

# One-Loop Renormalizable Wess-Zumino Model on Bosonic-Fermionic Noncommutative Superspace

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## Abstract

We construct a deformed Wess-Zumino model on the noncommutative superspace where the Bosonic and Fermionic coordinates are no longer commutative with each other. Using the background field method, we calculate the primary one-loop effective action based on the deformed action. By comparing the two actions, we find that the deformed Wess-Zumino model is not renormalizable. To obtain a renormalizable model, we combine the primary one-loop effective action with the deformed action, and then calculate the secondary one-loop effective action based on the combined action. After repeating this process to the third time, we finally give the one-loop renormalizable action up to the second order of Bosonic-Fermionic noncommutative parameters by using our specific techniques of calculation.

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# 1 Introduction

There are a lot of studies that generalize the ordinary spacetime to a noncommutative (NC) spacetime and provide interesting physical models [1, 2]. Because these studies rely only on a formal extension without experiments, there are many possible choices to construct NC spacetimes. A different kind of NC spacetimes usually gives different results that should generally not contradict with the very basic physical principle.

As an example, we take the canonical NC spacetime defined by

$$[x^\mu, x^\nu] = i\theta^{\mu\nu}, \quad \mu, \nu = 0, \dots, D-1, \quad (1)$$

where  $\theta^{\mu\nu}$  is a real antisymmetric constant tensor. In ref. [3] it is verified that a different choice of NC parameter  $\theta^{\mu\nu}$  changes the property of the model defined on it. When  $\theta^{0i} = 0$  the model is unitary, but non-unitary while  $\theta^{0i} \neq 0$ . The former can be obtained from string theory while the latter can not. In consequence, the fact that the model defined on the canonical NC spacetime with  $\theta^{0i} = 0$  is unitary results from the fact that string theory is unitary.

The above explanation is argued by, for instance, refs. [4, 5] where it is claimed that the breakdown of unitarity of the model defined on the canonical NC spacetime with  $\theta^{0i} \neq 0$  may have another origin, i.e., a wrong method to define the field theory on the NC spacetime may be utilized. The unitarity of the model can be restored if a different definition of field theory on the NC spacetime is given. Nonetheless, we note that it is not a generally accepted method to modify the definition of field theory on NC spacetimes.

Moreover, the ordinary spacetime can be extended to include Fermionic coordinates that are anticommutative to each other, which gives rise to a superspace [7, 8]. Therefore, one interesting generalization is a non-anticommutative (NAC) superspace. For example, in ref. [9] the  $\mathcal{N} = 1$  superspace is generalized to a NAC formulation. Based on the anticommutative relations of Fermionic coordinates, the algebraic relations of the other coordinates can be determined. Such a deformation from the anticommutative case to the non-anticommutative case is realized by using the star( $\star$ )-product associated with the NAC superspace. As a result, the Wess-Zumino model and the super Yang-Mills theory can consistently be defined on the NAC superspace.

Ref. [9] has inspired a lot of researches on the construction of models on the NAC superspace. In ref. [10] the renormalization property of the Wess-Zumino model on the NAC superspace is studied and new (non)renormalization theorems are proved. In addition, two global  $U(1)$  symmetries are found for the NAC Wess-Zumino model. In ref. [11] the NAC Wess-Zumino model is investigated by

calculating the 1PI effective action. At the one-loop order, new terms appear, which makes the model not renormalizable. To solve this problem, the new terms are added to the NAC Wess-Zumino action. With these new terms, the NAC Wess-Zumino model is proved to be renormalizable up to two loops. This work raises the problem that whether the NAC Wess-Zumino model can be renormalizable to higher order loop expansion. To answer this question, the dimensional analysis method is applied and the NAC Wess-Zumino model is thus proved [12] to be renormalizable to all loop orders in terms of the component form. Furthermore, the renormalizability can be retrieved [13] in terms of the superfield form. It should be emphasized that the key point of the proofs in refs. [12, 13] is the existence of the two global  $U(1)$  symmetries discovered in ref. [10].

Beyond an individual NC spacetime and an individual NAC superspace, the combination of a NC spacetime and a NAC superspace brings further interest, where the Bosonic-Fermionic noncommutative (BFNC) superspace has been considered by the argument of string theory [14]. On the BFNC superspace the Bosonic coordinates are no longer commutative with the Fermionic coordinates. A natural motivation is thus to study physical consequences of the BFNC superspace for some interesting models. To our knowledge, the properest choice is the Wess-Zumino model and the most appealing physical consequence among the many properties related to the model is renormalizability.

In this paper we construct the deformed Wess-Zumino model on the BFNC superspace and indeed work out its one-loop renormalizable action up to the second order of BFNC parameters. The paper is organized as follows. In section 2 we review the  $\mathcal{N} = 1$  supersymmetry and the Wess-Zumino model on the ordinary superspace in order to make this paper self-contained. In section 3 we derive the deformed Wess-Zumino action through substituting the ordinary product by the BFNC star( $\star$ )-product in the action of the (ordinary) Wess-Zumino model. For the purpose of searching a renormalizable Wess-Zumino action on the BFNC superspace in light of the background field method, we briefly review this method in section 4. In terms of our specific techniques of calculation, such as establishing the supersymmetry invariant subsets and their bases, we give the one-loop renormalizable action up to the second order of BFNC parameters in section 5. At last we present our conclusion and outlook in section 6. Incidentally, some useful formulae and the final long results are put into Appendices A and B.

## 2 $\mathcal{N} = 1$ Supersymmetry and Wess-Zumino Model

In this section we briefly review the supersymmetry [6] and the Wess-Zumino model on the ordinary superspace for the sake of making our discussions self-contained. We use the conventions given by Wess and Bagger [7].

The  $\mathcal{N} = 1$  superspace is represented by coordinates  $x^k$ ,  $\theta^\alpha$ , and  $\bar{\theta}^{\dot{\alpha}}$ , where  $k = 0, 1, 2, 3$ , and  $\alpha = 1, 2$ .  $x^k$  is a commutative number, while  $\theta^\alpha$  is an anticommutative number.

In the chiral coordinates  $y^k$ ,  $\theta^\alpha$ , and  $\bar{\theta}^{\dot{\alpha}}$ , where

$$y^k \equiv x^k + i\sigma^k_{\alpha\dot{\beta}}\theta^\alpha\bar{\theta}^{\dot{\beta}}, \quad (2)$$

$Q$  and  $D$  are defined by

$$\begin{aligned} Q_\alpha &\equiv \frac{\partial}{\partial\theta^\alpha}, & \bar{Q}_{\dot{\alpha}} &\equiv -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}} + 2i\theta^\beta\sigma^k_{\beta\dot{\alpha}}\frac{\partial}{\partial y^k}, \\ D_\alpha &\equiv \frac{\partial}{\partial\theta^\alpha} + 2i\sigma^k_{\alpha\dot{\beta}}\bar{\theta}^{\dot{\beta}}\frac{\partial}{\partial y^k}, & \bar{D}_{\dot{\alpha}} &\equiv -\frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}. \end{aligned} \quad (3)$$

The operators  $Q$  and  $D$  satisfy the following algebraic relations,

$$\begin{aligned} \{Q_\alpha, \bar{Q}_{\dot{\beta}}\} &= 2i\sigma^k_{\alpha\dot{\beta}}\partial_k, & \{Q_\alpha, Q_\beta\} &= \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0, \\ \{D_\alpha, \bar{D}_{\dot{\beta}}\} &= -2i\sigma^k_{\alpha\dot{\beta}}\partial_k, & \{D_\alpha, D_\beta\} &= \{\bar{D}_{\dot{\alpha}}, \bar{D}_{\dot{\beta}}\} = 0, \\ \{D_\alpha, Q_\beta\} &= \{D_\alpha, \bar{Q}_{\dot{\beta}}\} = \{\bar{D}_{\dot{\alpha}}, Q_\beta\} = \{\bar{D}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0, \end{aligned} \quad (4)$$

where the bracket of two operators, say  $O_1$  and  $O_2$ , is defined as  $\{O_1, O_2\} \equiv O_1O_2 + O_2O_1$ , and  $\partial_k \equiv \frac{\partial}{\partial y^k}$ .

The derivatives for anticommutative numbers take the forms,

$$\frac{\partial}{\partial\theta^\alpha}\theta^\beta = \delta_{\alpha\beta}, \quad \frac{\partial}{\partial\bar{\theta}^{\dot{\alpha}}}\bar{\theta}^{\dot{\beta}} = \delta_{\dot{\alpha}\dot{\beta}}, \quad (5)$$

where  $\delta_{\alpha\beta} = \delta_{\dot{\alpha}\dot{\beta}} = 1$  for  $\alpha = \beta$  and  $\dot{\alpha} = \dot{\beta}$ , and  $\delta_{\alpha\beta} = \delta_{\dot{\alpha}\dot{\beta}} = 0$ , for  $\alpha \neq \beta$  and  $\dot{\alpha} \neq \dot{\beta}$ .

### 2.1 Chiral Superfield

The chiral superfield  $\Phi$  and the antichiral superfields  $\Phi^+$  satisfy the following conditions, respectively,

$$\bar{D}_{\dot{\alpha}}\Phi = 0, \quad D_\alpha\Phi^+ = 0. \quad (6)$$

Here the chiral and antichiral superfields are defined by

$$\Phi \equiv \Phi(y, \theta), \quad \Phi^+ \equiv \Phi^+(y, \theta, \bar{\theta}), \quad (7)$$

which can be expressed by component fields as follows,

$$\begin{aligned}
\Phi(y, \theta) &= A(y) + \sqrt{2}\theta^\alpha\psi_\alpha(y) + \theta^\alpha\theta_\alpha F(y), \\
\Phi^+(y, \theta, \bar{\theta}) &= A^*(y) + \sqrt{2}\bar{\theta}_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}}(y) + \bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}F^*(y) - 2i\sigma^k_{\alpha\dot{\beta}}\theta^\alpha\bar{\theta}^{\dot{\beta}}\partial_k A^*(y) \\
&\quad + i\sqrt{2}\sigma^k_{\alpha\dot{\beta}}\theta^\alpha\bar{\theta}_{\dot{\gamma}}\bar{\theta}^{\dot{\gamma}}\partial_k\bar{\psi}^{\dot{\beta}}(y) + \eta^{kl}\theta^\alpha\theta_\alpha\bar{\theta}_{\dot{\beta}}\bar{\theta}^{\dot{\beta}}\partial_k\partial_l A^*(y),
\end{aligned} \tag{8}$$

where  $A(y)$  and  $F(y)$  are scalar fields,  $\psi_\alpha(y)$  and  $\bar{\psi}^{\dot{\alpha}}(y)$  are spinor fields.

Spinor indices are raised and lowered by using the antisymmetric symbols  $\epsilon^{\alpha\beta}$  and  $\epsilon_{\alpha\beta}$ , respectively,

$$\psi^\alpha \equiv \epsilon^{\alpha\beta}\psi_\beta, \quad \psi_\alpha \equiv \epsilon_{\alpha\beta}\psi^\beta, \tag{9}$$

where the antisymmetric symbols satisfy the relation:  $\epsilon_{\alpha\beta}\epsilon^{\beta\gamma} = \delta_{\alpha\gamma}$ .

In the following context, the abbreviations

$$\theta^2 \equiv \theta^\alpha\theta_\alpha, \quad \bar{\theta}^2 \equiv \bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}} \tag{10}$$

are used.

## 2.2 Supersymmetry Transformation

The supersymmetry transformation of the superfield  $\mathcal{F}$  is defined as

$$\delta_\xi \mathcal{F} \equiv \left( \xi^\alpha Q_\alpha - \bar{\xi}^{\dot{\beta}} \bar{Q}_{\dot{\beta}} \right) \mathcal{F}, \tag{11}$$

where  $\xi^\alpha$  and  $\bar{\xi}^{\dot{\beta}}$  are spinor parameters.

Applying the above transformation rule to the chiral superfield, one can obtain the transformations of component fields,

$$\begin{aligned}
\delta_\xi A &= \sqrt{2}\xi^\alpha\psi_\alpha, \\
\delta_\xi \psi_\alpha &= \sqrt{2}\xi_\alpha F + i\sqrt{2}\sigma^k_{\alpha\dot{\beta}}\bar{\xi}^{\dot{\beta}}\partial_k A, \\
\delta_\xi F &= i\sqrt{2}\sigma^k_{\alpha\dot{\beta}}\bar{\xi}^{\dot{\beta}}\partial_k\psi^\alpha.
\end{aligned} \tag{12}$$

Note that in eq. (11) both the superfield  $\mathcal{F}$  and the operator  $Q$  are expressed in the chiral coordinates  $y$ ,  $\theta$ , and  $\bar{\theta}$  (see eqs. (2) and (3)).

## 2.3 Wess-Zumino Model

The action of the Wess-Zumino model is

$$\mathcal{S}_{\text{WZ}} = \int d^4x \left\{ \Phi^+\Phi|_{\theta^2\bar{\theta}^2} + \frac{m}{2}\Phi\Phi|_{\theta^2} + \frac{g}{3}\Phi\Phi\Phi|_{\theta^2} + \frac{m}{2}\Phi^+\Phi^+|_{\bar{\theta}^2} + \frac{g}{3}\Phi^+\Phi^+\Phi^+|_{\bar{\theta}^2} \right\}, \tag{13}$$

where  $\Phi$  and  $\Phi^+$  are given by eq. (8),  $\Phi^+\Phi|_{\theta^2\bar{\theta}^2}$  denotes the  $\theta^2\bar{\theta}^2$  component of  $\Phi^+\Phi$ , and the other terms have the similar meaning.

The Wess-Zumino action eq. (13) can be transformed to a total superspace integral,

$$\begin{aligned} \mathcal{S}_{\text{WZ}} = \int d^8z \left\{ \Phi^+\Phi - \frac{m}{8}\Phi\left(\frac{D^2}{\square}\Phi\right) - \frac{m}{8}\Phi^+\left(\frac{\bar{D}^2}{\square}\Phi^+\right) \right. \\ \left. - \frac{g}{12}\Phi\Phi\left(\frac{D^2}{\square}\Phi\right) - \frac{g}{12}\Phi^+\Phi^+\left(\frac{\bar{D}^2}{\square}\Phi^+\right) \right\}, \end{aligned} \quad (14)$$

where  $d^8z \equiv d^4x d^2\theta d^2\bar{\theta}$ ,  $D^2 = \epsilon^{\alpha\beta} D_\beta D_\alpha$ , and  $\bar{D}^2 = \epsilon^{\dot{\alpha}\dot{\beta}} \bar{D}_{\dot{\alpha}} \bar{D}_{\dot{\beta}}$ .

### 3 Deformed Action from BFNC $\star$ -Product

In this section we at first give the  $\star$ -product of the BFNC superspace and its corresponding algebraic relations of coordinates, and then define the deformed Wess-Zumino model in the component form on the Euclidean BFNC superspace. For calculating effective actions in section 5 we further transform the deformed model into its superfield form. In the following the Lorentz signatures are still used as pointed out by Seiberg [9].

#### 3.1 BFNC $\star$ -Product

The BFNC  $\star$ -product can be expressed in terms of the tensor algebraic notation which is frequently used in quantum group theory [15],

$$\mathbf{F} \star \mathbf{G} \equiv \mu \left\{ \exp \left[ \frac{i}{2} \Lambda^{k\alpha} \left( \frac{\partial}{\partial y^k} \otimes \frac{\partial}{\partial \theta^\alpha} - \frac{\partial}{\partial \theta^\alpha} \otimes \frac{\partial}{\partial y^k} \right) \right] \triangleright (\mathbf{F} \otimes \mathbf{G}) \right\}, \quad (15)$$

where  $\Lambda^{k\alpha}$  denotes a BFNC parameter,  $k = 0, 1, 2, 3$ , and  $\alpha = 1, 2$ , which implies that there are eight independent BFNC parameter components in total.

For the notation in eq. (15), the tensor product is defined as

$$(A \otimes B)(C \otimes D) = (-1)^{|B||C|} AC \otimes BD, \quad (16)$$

where  $|B|$  is the grade of  $B$  that equals 1 for a Bosonic element and  $-1$  for a Fermionic element. The symbol  $\triangleright$  represents an action of an operator on a function, and  $\mu$  denotes the change from the tensor product  $\otimes$  to an ordinary product, and  $\mathbf{F}$  and  $\mathbf{G}$  stand for any superfields.

The Taylor expansion of eq. (15) takes the form,

$$\mathbf{F} \star \mathbf{G} = \mathbf{F}\mathbf{G} - \frac{i}{2} \Lambda^{k\alpha} (\partial_\alpha \mathbf{F}) (\partial_k \mathbf{G}) + (-1)^{|\mathbf{F}|} \frac{i}{2} \Lambda^{k\alpha} (\partial_k \mathbf{F}) (\partial_\alpha \mathbf{G})$$



$$\begin{aligned}
& +\frac{1}{8}\Lambda^{k\alpha}\Lambda^{l\beta}(\partial_k\partial_l\mathbf{F})(\partial_\alpha\partial_\beta\mathbf{G})+\frac{1}{8}\Lambda^{k\alpha}\Lambda^{l\beta}(\partial_\alpha\partial_\beta\mathbf{F})(\partial_k\partial_l\mathbf{G}) \\
& +(-1)^{|\mathbf{F}|}\frac{1}{4}\Lambda^{k\alpha}\Lambda^{l\beta}(\partial_\beta\partial_k\mathbf{F})(\partial_\alpha\partial_l\mathbf{G}) \\
& -\frac{i}{16}\Lambda^{k\alpha}\Lambda^{l\beta}\Lambda^{m\zeta}(\partial_\alpha\partial_l\partial_m\mathbf{F})(\partial_\beta\partial_\zeta\partial_k\mathbf{G}) \\
& +(-1)^{|\mathbf{F}|}\frac{i}{16}\Lambda^{k\alpha}\Lambda^{l\beta}\Lambda^{m\zeta}(\partial_\alpha\partial_\beta\partial_m\mathbf{F})(\partial_\zeta\partial_k\partial_l\mathbf{G}) \\
& -\frac{1}{64}\Lambda^{k\alpha}\Lambda^{l\beta}\Lambda^{m\zeta}\Lambda^{n\iota}(\partial_\alpha\partial_\zeta\partial_l\partial_n\mathbf{F})(\partial_\beta\partial_\iota\partial_k\partial_m\mathbf{G}), \tag{17}
\end{aligned}$$

where  $\partial_k \equiv \frac{\partial}{\partial y^k}$ , and  $\partial_\alpha \equiv \frac{\partial}{\partial \theta^\alpha}$ .

As a simple application of the  $\star$ -product eq.(17), we calculate some  $\star$ -algebraic relations of coordinates,

$$\begin{aligned}
\{y^k, \theta^\alpha\}_\star &= i\Lambda^{k\alpha}, \quad [x^k, y^l]_\star = -\sigma^k_{\alpha\dot{\beta}}\Lambda^{l\alpha}\bar{\theta}^{\dot{\beta}}, \quad [x^k, \theta^\alpha]_\star = i\Lambda^{k\alpha}, \\
[x^k, x^l]_\star &= \sigma^l_{\alpha\dot{\beta}}\Lambda^{k\alpha}\bar{\theta}^{\dot{\beta}} - \sigma^k_{\alpha\dot{\beta}}\Lambda^{l\alpha}\bar{\theta}^{\dot{\beta}}, \tag{18}
\end{aligned}$$

where the algebraic relations in eq. (4) are not modified.

### 3.2 Deformed Wess-Zumino Model

To analyze the effect of the BFNC superspace on the Wess-Zumino model, we replace the ordinary product in eq. (13) by the  $\star$ -product defined by eq. (15) and give the deformed action,

$$\mathcal{S}_{\text{NC}} \equiv \int d^4x \left\{ \Phi^+ \star \Phi|_{\theta^2\bar{\theta}^2} + \frac{m}{2}\Phi \star \Phi|_{\theta^2} + \frac{g}{3}\Phi \star \Phi \star \Phi|_{\theta^2} + \frac{m}{2}\Phi^+ \star \Phi^+|_{\bar{\theta}^2} + \frac{g}{3}\Phi^+ \star \Phi^+ \star \Phi^+|_{\bar{\theta}^2} \right\}, \tag{19}$$

where the chiral superfield  $\Phi$  and the antichiral superfield  $\Phi^+$  are given in the chiral coordinates as in eq. (8).

To obtain the explicit form of the deformed Wess-Zumino action, we calculate the  $\star$ -product of the chiral and antichiral superfields,

$$\begin{aligned}
\int d^4x \Phi^+ \star \Phi|_{\theta^2\bar{\theta}^2} &= \int d^4x \left\{ -i\sigma^k_{\alpha\dot{\beta}}\psi^\alpha\partial_k\bar{\psi}^{\dot{\beta}} + A\Box A^* + FF^* \right\}, \\
\int d^4x \Phi \star \Phi|_{\theta^2} &= \int d^4x \{ 2AF - \psi^\alpha\psi_\alpha \}, \\
\int d^4x \Phi^+ \star \Phi^+|_{\bar{\theta}^2} &= \int d^4x \{ -\bar{\psi}_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}} + 2A^*F^* \}, \\
\int d^4x \Phi \star \Phi \star \Phi|_{\theta^2} &= \int d^4x \left\{ -3\psi^\alpha\psi_\alpha A + 3AAF - \frac{3}{4}\Lambda^{kl}\partial_k\partial_l A F F \right. \\
&\quad \left. + \frac{3}{2}\Lambda^{k\alpha}\Lambda^{l\beta}\partial_k\psi_\beta\partial_l\psi_\alpha F + \frac{1}{16}\Lambda^{kl}\Lambda^{no}F\partial_k\partial_l F\partial_n\partial_o F \right\},
\end{aligned}$$

$$\begin{aligned}
\int d^4x \Phi^+ \star \Phi^+ \star \Phi^+ |_{\bar{\theta}^2} &= \int d^4x \left\{ -3\bar{\psi}_{\dot{\alpha}}\bar{\psi}^{\dot{\alpha}}A^* + 3A^*A^*F^* - \frac{1}{2}\Lambda^{kl}A^*\square A^*\partial_k\partial_l A^* \right. \\
&\quad + \frac{1}{2}\eta^{kl}\Lambda^{no}A^*\partial_k\partial_n A^*\partial_l\partial_o A^* \\
&\quad \left. + (\sigma^{kl})_{\alpha}^{\beta}\Lambda^{n\alpha}\Lambda^o_{\beta}A^*\partial_k\partial_n A^*\partial_l\partial_o A^* \right\}, \tag{20}
\end{aligned}$$

where  $\Lambda^{kl} \equiv \epsilon^{\alpha\beta}\Lambda^k_{\beta}\Lambda^l_{\alpha}$ , and the various identities given in eq. (A1) have been used.

Now we make the transformation for the deformed action from its component form eq. (20) to the desired superfield form. This performance is necessary for us to compute effective actions in section 5.

The component fields can be transformed to the chiral and antichiral superfields as follows,

$$\begin{aligned}
A &= \Phi|, & \psi_{\alpha} &= \frac{1}{\sqrt{2}}D_{\alpha}\Phi|, & F &= -\frac{1}{4}(D^2\Phi)|, \\
A^* &= \Phi^+|, & \bar{\psi}_{\dot{\alpha}} &= \frac{1}{\sqrt{2}}\bar{D}_{\dot{\alpha}}\Phi^+|, & F^* &= -\frac{1}{4}(\bar{D}^2\Phi^+)|, \tag{21}
\end{aligned}$$

where the symbol  $|$  represents setting all  $\theta^{\alpha}$  and  $\bar{\theta}^{\dot{\alpha}}$  be zero.

The deformed action  $\mathcal{S}_{\text{NC}}$  is the sum of the ordinary part  $\mathcal{S}_{\text{WZ}}$  (the terms in the first two lines, also see eq. (14)) and the noncommutative part  $\mathcal{S}_{\Lambda}$  (the remaining terms),

$$\begin{aligned}
\mathcal{S}_{\text{NC}} &= \int d^8z \left\{ \Phi^+\Phi - \frac{m}{8}\Phi\left(\frac{D^2}{\square}\Phi\right) - \frac{m}{8}\Phi^+\left(\frac{\bar{D}^2}{\square}\Phi^+\right) \right. \\
&\quad - \frac{g}{12}\Phi\Phi\left(\frac{D^2}{\square}\Phi\right) - \frac{g}{12}\Phi^+\Phi^+\left(\frac{\bar{D}^2}{\square}\Phi^+\right) \\
&\quad + \frac{1}{3072}(-g)\Lambda^{kl}\Lambda^{no}\theta^4(D^2\Phi)\partial_l\partial_k(D^2\Phi)\partial_o\partial_n(D^2\Phi) \\
&\quad + \frac{1}{32}(-g)\Lambda^{kl}\theta^4\Phi(D^2\Phi)\partial_l\partial_k(D^2\Phi) \\
&\quad + \frac{1}{32}(-g)\Lambda^{kl}\theta^4\Phi\partial_k(D^2\Phi)\partial_l(D^2\Phi) \\
&\quad + \frac{1}{6}(-g)\Lambda^{kl}\theta^4\Phi^+\square\Phi^+\partial_k\partial_l\Phi^+ \\
&\quad + \frac{1}{3}(-g)(\sigma\Lambda\Lambda^{kl})^{no}\theta^4\Phi^+\partial_k\partial_n\Phi^+\partial_l\partial_o\Phi^+ \\
&\quad + \frac{1}{6}g\eta^{kl}\Lambda^{no}\theta^4\Phi^+\partial_k\partial_n\Phi^+\partial_l\partial_o\Phi^+ \\
&\quad + \frac{1}{16}(-g)\epsilon^{\alpha\beta}\Lambda^{kl}\theta^4\partial_k(D_{\alpha}\Phi)\partial_l(D_{\beta}\Phi)(D^2\Phi) \\
&\quad \left. + \frac{1}{16}(-g)\epsilon^{\alpha\beta}\epsilon^{\zeta\iota}\Lambda^k_{\beta}\Lambda^l_{\iota}\theta^4\partial_k(D_{\alpha}\Phi)\partial_l(D_{\zeta}\Phi)(D^2\Phi) \right\}, \tag{22}
\end{aligned}$$

where symbol  $\theta^4$  has been introduced in order to express the above equation concisely,

$$\theta^4 \equiv \theta^2\bar{\theta}^2, \tag{23}$$

and the meanings of  $\theta^2$  and  $\bar{\theta}^2$  are provided in eq. (10).

Note that  $\mathcal{S}_{\text{NC}}$  (eq. (22)) contains the second and fourth orders of contributions of the BFNC parameters  $\Lambda^{k\alpha}$ 's. In addition, it is invariant under the 1/2 supersymmetry transformation defined by

$$\delta_\xi \Phi \equiv \xi^\alpha Q_\alpha \Phi, \quad \delta_\xi \Phi^+ \equiv \xi^\alpha Q_\alpha \Phi^+. \quad (24)$$

## 4 Background Field Method

In this section we give a brief review on the background field method [8, 16]. The purpose is to prepare some necessary formulae, such as the matrices  $M$ ,  $M^{-1}$ , and  $V$ , for deriving effective actions related to the deformed Wess-Zumino action  $\mathcal{S}_{\text{NC}}$  (eq. (22)) on the BFNC superspace in section 5.

### 4.1 General Procedure

We split the chiral and antichiral superfields into the classical and quantum parts,

$$\Phi \rightarrow \Phi + \Phi_q, \quad \Phi^+ \rightarrow \Phi^+ + \Phi_q^+, \quad (25)$$

where the quantum parts of the chiral and antichiral superfields satisfy the constraints:  $\bar{D}_{\dot{\alpha}} \Phi_q = D_\alpha \Phi_q^+ = 0$ .

To integrate out the quantum parts we represent them by the general superfields  $\Sigma$  and  $\Sigma^+$ , respectively,

$$\Phi_q = -\frac{1}{4} \bar{D}^2 \Sigma, \quad \Phi_q^+ = -\frac{1}{4} D^2 \Sigma^+. \quad (26)$$

Then the quantum fields  $\Sigma$  and  $\Sigma^+$  can be treated as free fields.

Because of the introduction of the covariant derivative in eq. (26) and the nilpotency of operators  $D$  and  $\bar{D}$ , we have the new gauge symmetry,

$$\Sigma \rightarrow \Sigma + \bar{D}_{\dot{\alpha}} \bar{\Lambda}^{\dot{\alpha}}, \quad \Sigma^+ \rightarrow \Sigma^+ + D^\alpha \Lambda_\alpha, \quad (27)$$

and thus need to add the gauge fixing action  $\mathcal{S}_{\text{GF}}$ ,

$$\mathcal{S}_{\text{GF}} = \int d^8 z \left\{ -\frac{3}{16} \xi \epsilon^{\dot{\alpha}\dot{\beta}} (\bar{D}_{\dot{\alpha}} \Sigma^+) (\bar{D}_{\dot{\beta}} D^2 \Sigma) - \frac{1}{4} \xi \epsilon^{\alpha\beta} \epsilon^{\dot{\alpha}\dot{\beta}} (\bar{D}_{\dot{\alpha}} \Sigma^+) (D_\beta \bar{D}_{\dot{\beta}} D_\alpha \Sigma) \right\}, \quad (28)$$

in order to eliminate the degree of freedom in eq. (27), where  $\xi$  is a gauge fixing parameter.

Replacing the chiral and antichiral superfields in the deformed Wess-Zumino action  $\mathcal{S}_{\text{NC}}$  (eq. (22)) by eq. (25) and keeping only the quadratic part of the quantum fields in the combined action  $\mathcal{S}_{\text{NC}} + \mathcal{S}_{\text{GF}}$ , we have

$$\mathcal{S}^{(2)} = \frac{1}{2} \int d^8 z \begin{pmatrix} \Sigma & \Sigma^+ \end{pmatrix} (M + V) \begin{pmatrix} \Sigma \\ \Sigma^+ \end{pmatrix}, \quad (29)$$

where the matrices  $M$  and  $V$  correspond to the kinetic and interacting parts of the action  $\mathcal{S}_{\text{NC}} + \mathcal{S}_{\text{GF}}$ .

After abstracting the free part in the action  $\mathcal{S}^{(2)}$  and making the Taylor expansion, we obtain the one-loop  $n$ -point effective action  $\Gamma^{(n)}$  from the following formula,

$$\Gamma = \frac{i}{2} \text{STr} \ln (1 + M^{-1}V) = \frac{i}{2} \sum_{n=1}^{\infty} \text{STr} \left[ \frac{(-1)^{n+1}}{n} (M^{-1}V)^n \right] \equiv \sum_{n=1}^{\infty} \Gamma^{(n)}. \quad (30)$$

## 4.2 Calculation of $M$ and $V$

Each term in  $\mathcal{S}^{(2)}$  can be represented in a general form,

$$\int d^8z (\partial_1 X) Z (\partial_2 Y), \quad (31)$$

where  $\partial_1$  and  $\partial_2$  stand for any products of the following operators,

$$\partial_k, \quad D_\alpha, \quad \bar{D}_{\dot{\alpha}}, \quad \square^{-1}, \quad (32)$$

and  $X$  and  $Y$  denote  $\Sigma$  or  $\Sigma^+$ ,  $Z$  can be the identity operator,  $\theta^4$ , or a function of chiral and antichiral superfields  $\Phi$  and  $\Phi^+$ .

To obtain  $M + V$ , we find the following rules for each term in  $\mathcal{S}^{(2)}$ ,

- Each  $\int d^8z (\partial_1 \Sigma) Z (\partial_2 \Sigma)$  contributes  $(\partial_1^T Z \partial_2 + (\partial_1^T Z \partial_2)^T)$  to  $(M + V)_{1,1}$ ,
- Each  $\int d^8z (\partial_1 \Sigma) Z (\partial_2 \Sigma^+)$  contributes  $\partial_1^T Z \partial_2$  to  $(M + V)_{1,2}$ , and  $(\partial_1^T Z \partial_2)^T$  to  $(M + V)_{2,1}$ ,
- Each  $\int d^8z (\partial_1 \Sigma^+) Z (\partial_2 \Sigma^+)$  contributes  $(\partial_1^T Z \partial_2 + (\partial_1^T Z \partial_2)^T)$  to  $(M + V)_{2,2}$ ,

where  $(M + V)_{i,j}$  is the  $(i, j)$  component of matrix  $M + V$ . With these rules we can ensure that the matrices  $M$  and  $V$  are symmetric, that is, they satisfy  $M = M^T$  and  $V = V^T$ .

For any operators  $A$  and  $B$ , the transposition of their product is defined by

$$(AB)^T \equiv (-1)^{|A||B|} B^T A^T, \quad (33)$$

and the action of  $T$  on operators is defined as follows:

$$\begin{aligned} \partial_k^T &\equiv -\partial_k, & \square^T &\equiv \square, & \left(\frac{1}{\square}\right)^T &\equiv \frac{1}{\square}, \\ \partial_\alpha^T &\equiv -\partial_\alpha, & \bar{\partial}_{\dot{\alpha}}^T &\equiv -\bar{\partial}_{\dot{\alpha}}, \\ D_\alpha^T &\equiv -D_\alpha, & \bar{D}_{\dot{\alpha}}^T &\equiv -\bar{D}_{\dot{\alpha}}, \\ (D^2)^T &\equiv D^2, & (\bar{D}^2)^T &\equiv \bar{D}^2. \end{aligned} \quad (34)$$

In accordance with the above considerations, we can easily obtain  $M$  and  $V$  from  $M + V$ , where  $M$  does not include  $\Phi$  and  $\Phi^+$ , but  $V$  contains them. The matrix  $M$  takes the form,

$$\begin{aligned}
M_{1,1} &= -\frac{1}{4}m\bar{D}^2, \\
M_{1,2} &= \frac{1}{16}\bar{D}^2D^2 + \frac{3}{16}\xi D^2\bar{D}^2 + \frac{1}{4}\xi\epsilon^{\alpha\beta}\epsilon^{\dot{\zeta}i}D_\alpha\bar{D}_iD_\beta\bar{D}_{\dot{\zeta}}, \\
M_{2,1} &= \frac{3}{16}\xi\bar{D}^2D^2 + \frac{1}{16}D^2\bar{D}^2 + \frac{1}{4}\xi\epsilon^{\alpha\beta}\epsilon^{\dot{\zeta}i}\bar{D}_{\dot{\zeta}}D_\beta\bar{D}_iD_\alpha, \\
M_{2,2} &= -\frac{1}{4}mD^2,
\end{aligned} \tag{35}$$

and matrix  $V$  is

$$\begin{aligned}
V_{1,2} &= 0, \quad V_{2,1} = 0, \\
V_{1,1} &= \frac{1}{2}(-g)\Phi\bar{D}^2 + \frac{1}{512}(-g)\bar{D}^2D^2\theta^4(D^2\Phi)p\Lambda^2\bar{D}^2 + \frac{1}{512}(-g)p\Lambda^2\bar{D}^2\theta^4(D^2\Phi)D^2\bar{D}^2 \\
&\quad + \frac{1}{128}(-g)\epsilon^{\alpha\beta}\epsilon^{\dot{\zeta}i}p\Lambda_\beta\bar{D}^2D_i\theta^4(D^2\Phi)p\Lambda_\zeta D_\alpha\bar{D}^2 + \frac{1}{8192}(-g)\bar{D}^2D^2\theta^4(p\Lambda^2(D^2\Phi))p\Lambda^2D^2\bar{D}^2 \\
&\quad + \frac{1}{128}(-g)\epsilon^{\alpha\beta}\epsilon^{\dot{\zeta}i}\bar{D}^2D^2\theta^4(p\Lambda_\beta(D_i\Phi))p\Lambda_\zeta D_\alpha\bar{D}^2 + \frac{1}{512}(-g)\bar{D}^2D^2\theta^4(p\Lambda^2\Phi)D^2\bar{D}^2 \\
&\quad + \frac{1}{128}(-g)\epsilon^{\alpha\beta}\epsilon^{\dot{\zeta}i}p\Lambda_\beta\bar{D}^2D_i\theta^4(p\Lambda_\zeta(D_\alpha\Phi))D^2\bar{D}^2, \\
V_{2,2} &= \frac{1}{2}(-g)\Phi^+D^2 + \frac{1}{96}(-g)D^2\theta^4(\square\Phi^+)p\Lambda^2D^2 + \frac{1}{96}(-g)p\Lambda^2D^2\theta^4(\square\Phi^+)D^2 \\
&\quad + \frac{1}{96}(-g)\square D^2\theta^4\Phi^+p\Lambda^2D^2 + \frac{1}{96}(-g)p\Lambda^2D^2\theta^4\Phi^+\square D^2 + \frac{1}{96}(-g)D^2\theta^4(p\Lambda^2\Phi^+)\square D^2 \\
&\quad + \frac{1}{48}(-g)\epsilon^{\alpha\beta}D^2\theta^4(p\sigma_{\beta\dot{\zeta}}(p\Lambda_{\dot{\zeta}}\Phi^+))\overline{p\sigma}^{\dot{\zeta}i}p\Lambda_\alpha D^2 + \frac{1}{48}(-g)\epsilon^{\alpha\beta}p\sigma_{\beta\dot{\zeta}}p\Lambda_{\dot{\zeta}}D^2\theta^4\Phi^+\overline{p\sigma}^{\dot{\zeta}i}p\Lambda_\alpha D^2 \\
&\quad + \frac{1}{96}(-g)\square D^2\theta^4(p\Lambda^2\Phi^+)D^2 + \frac{1}{48}(-g)\epsilon^{\alpha\beta}p\sigma_{\beta\dot{\zeta}}p\Lambda_{\dot{\zeta}}D^2\theta^4(\overline{p\sigma}^{\dot{\zeta}i}(p\Lambda_\alpha\Phi^+))D^2,
\end{aligned} \tag{36}$$

where the following new symbols have been introduced in order to express matrix  $V$  concisely,

$$\begin{aligned}
p\sigma_{\alpha\dot{\beta}} &\equiv \sigma^k_{\alpha\dot{\beta}}\partial_k, & \overline{p\sigma}^{\dot{\alpha}\beta} &\equiv (\bar{\sigma}^k)^{\dot{\alpha}\beta}\partial_k, \\
p\Lambda^\alpha &\equiv \Lambda^{k\alpha}\partial_k, & p\Lambda_\alpha &\equiv \Lambda^k_\alpha\partial_k, \\
p\Lambda^2 &\equiv p\Lambda^\alpha p\Lambda_\alpha.
\end{aligned} \tag{37}$$

### 4.3 Calculation of $M^{-1}$

We at first briefly review the general procedure to calculate  $M^{-1}$  [7]. For a general  $2 \times 2$  matrix  $X$  that can be written as

$$X = aP_1 + dP_2 + bP_+ + cP_- + eP_T, \tag{38}$$

where  $a, b, c, d$ , and  $e$  are  $2 \times 2$  coefficient matrices, and  $P_1, P_2, P_+, P_-$ , and  $P_T$  are related to derivatives of the chiral coordinates and defined by

$$P_1 \equiv \frac{1}{16}\square^{-1}D^2\bar{D}^2, \quad P_2 \equiv \frac{1}{16}\square^{-1}\bar{D}^2D^2,$$

$$\begin{aligned}
P_+ &\equiv \frac{1}{4}\square^{-\frac{1}{2}}D^2, & P_- &\equiv \frac{1}{4}\square^{-\frac{1}{2}}\bar{D}^2, \\
P_T &\equiv -\frac{1}{8}\epsilon^{\alpha\beta}\square^{-1}D_\beta\bar{D}^2D_\alpha,
\end{aligned} \tag{39}$$

one has the inverse of  $X$ ,

$$\begin{aligned}
X^{-1} &= (a - bd^{-1}c)^{-1}P_1 + (d - ca^{-1}b)^{-1}P_2 - a^{-1}b(d - ca^{-1}b)^{-1}P_+ \\
&\quad - d^{-1}c(a - bd^{-1}c)^{-1}P_- + e^{-1}P_T,
\end{aligned} \tag{40}$$

if  $a$ ,  $d$ , and  $e$  are invertible.

Now we turn to our case and at first transform  $M$  (eq. (35)) to the following form,

$$M = \begin{pmatrix} -m\square^{\frac{1}{2}}P_- & \xi\square P_1 + \square P_2 + \xi\square P_T \\ \square P_1 + \xi\square P_2 + \xi\square P_T & -m\square^{\frac{1}{2}}P_+ \end{pmatrix}, \tag{41}$$

where we have used the equalities

$$\begin{aligned}
\epsilon^{\alpha\gamma}\epsilon^{\dot{\beta}\dot{\zeta}}D_\alpha\bar{D}_{\dot{\beta}}D_\gamma\bar{D}_{\dot{\zeta}} &= \frac{1}{2}D^2\bar{D}^2 - \frac{1}{2}\epsilon^{\alpha\beta}D_\alpha\bar{D}^2D_\beta, \\
\epsilon^{\beta\zeta}\epsilon^{\dot{\alpha}\dot{\gamma}}\bar{D}_{\dot{\alpha}}D_\beta\bar{D}_{\dot{\gamma}}D_\zeta &= \frac{1}{2}\bar{D}^2D^2 - \frac{1}{2}\epsilon^{\alpha\beta}D_\alpha\bar{D}^2D_\beta.
\end{aligned} \tag{42}$$

Next, by applying eq. (40) to eq. (41) we obtain the inverse of  $M$ ,

$$\begin{aligned}
(M^{-1})_{1,1} &= \frac{1}{4}\frac{m}{\square - m^2}\square^{-1}D^2, \\
(M^{-1})_{1,2} &= \frac{1}{16\xi}\square^{-2}\bar{D}^2D^2 + \frac{1}{16}\frac{1}{\square - m^2}\square^{-1}D^2\bar{D}^2 - \frac{1}{8\xi}\epsilon^{\alpha\beta}\square^{-2}D_\beta\bar{D}^2D_\alpha, \\
(M^{-1})_{2,1} &= \frac{1}{16}\frac{1}{\square - m^2}\square^{-1}\bar{D}^2D^2 + \frac{1}{16\xi}\square^{-2}D^2\bar{D}^2 - \frac{1}{8\xi}\epsilon^{\alpha\beta}\square^{-2}D_\beta\bar{D}^2D_\alpha, \\
(M^{-1})_{2,2} &= \frac{1}{4}\frac{m}{\square - m^2}\square^{-1}\bar{D}^2.
\end{aligned} \tag{43}$$

## 5 Main Result

### 5.1 General Procedure for Calculating Supertrace

We give the general procedure to compute  $\text{STr}$ . The  $n$ -point function corresponds to

$$\frac{i(-1)^{n+1}}{2n}\text{STr}(M^{-1}V)^n, \tag{44}$$

where  $\text{STr}$  is the supertrace. The whole process can be divided into two parts, the first is to calculate the trace for the spinor coordinate  $\theta^4 = \theta^2\bar{\theta}^2$ , and the second part is to calculate the trace for the Bosonic coordinate  $x^k$ .

Except for the BFNC parameter factors, the general form of terms in  $M^{-1}V$  is

$$\mathcal{D}_A \partial_C \frac{1}{\square - m^2} (\square^{-1})^s \theta^4 \mathcal{F}_B \partial_{C'} \mathcal{D}_{A'}, \quad (45)$$

where  $s = 0$  in some terms which means  $\square^{-1}$  does not appear, or  $s = 1$  in some terms which means  $\square^{-1}$  appears. We explain the meanings of the symbols in eq. (45) as follows.

$\mathcal{D}_A$  and  $\mathcal{D}_{A'}$  stand for any elements of set  $\mathcal{D}$ ,

$$\mathcal{D} = \{D_\alpha, \bar{D}_{\dot{\beta}}, D^2, \bar{D}^2, D_\alpha \bar{D}_{\dot{\beta}}, D^2 \bar{D}_{\dot{\beta}}, D_\alpha \bar{D}^2, D^2 \bar{D}^2\}. \quad (46)$$

To obtain the set  $\mathcal{D}$ , we have taken into account the  $D$  algebraic relations eq. (A2) and move every  $D_\alpha$  to the left side of  $\bar{D}_{\dot{\beta}}$ .

$\partial_C$  and  $\partial_{C'}$  denote any elements of set  $\partial$ ,

$$\partial = \{\partial_k, \partial_k \partial_l, \partial_k \partial_l \partial_m, \dots\}. \quad (47)$$

$\mathcal{F}_B$  is any elements of set  $\mathcal{F}$ ,

$$\mathcal{F} = \{((\partial)^n \Phi), ((\partial)^n D_\alpha \Phi), ((\partial)^n D^2 \Phi), ((\partial)^n \Phi^+), ((\partial)^n \bar{D}_\alpha \Phi^+), ((\partial)^n \bar{D}^2 \Phi^+)\}, \quad (48)$$

where  $(\partial)^n$  implies the product of  $n$   $\partial_k$ 's,

$$(\partial)^n = \partial_{k_1} \partial_{k_2} \dots \partial_{k_n}, \quad (49)$$

and  $n$  is a non-negative integer. To obtain  $\mathcal{F}$ , we have used the  $D$  algebraic relations eq. (A2) and the chiral and antichiral conditions eq. (6).

From eq. (45), we can determine the general form of eq. (44),

$$\text{STr} \left\{ \underbrace{\left( \mathcal{D}_{A_1} \partial_{C_1} \frac{1}{\square - m^2} (\square^{-1})^{s_1} \theta^4 \mathcal{F}_{B_1} \partial_{C'_1} \mathcal{D}_{A'_1} \right) \cdots \left( \mathcal{D}_{A_n} \partial_{C_n} \frac{1}{\square - m^2} (\square^{-1})^{s_n} \theta^4 \mathcal{F}_{B_n} \partial_{C'_n} \mathcal{D}_{A'_n} \right)}_{n \text{ terms}} \right\}, \quad (50)$$

where  $\mathcal{D}_{A_i}$  and  $\mathcal{D}_{A'_i}$  stand for any elements of set  $\mathcal{D}$ ,  $\mathcal{F}_{B_i}$  means any elements of set  $\mathcal{F}$ ,  $\partial_{C_i}$  and  $\partial_{C'_i}$  denote any elements of set  $\partial$ , and  $s_i$  takes the value of 0 or 1,  $i = 1, 2, \dots, n$ .

We use the following procedure to show how to calculate eq. (50), for simplicity, but without the loss of generality, we take  $n = 2$  as a sample.

*Step 1:* By using the symmetry of supertrace  $\text{STr}$ ,

$$\text{STr}\{\mathcal{X}\mathcal{Y}\} = (-1)^{|\mathcal{X}||\mathcal{Y}|} \text{STr}\{\mathcal{Y}\mathcal{X}\}, \quad (51)$$

where  $\mathcal{X}$  and  $\mathcal{Y}$  are any operators, we move  $\partial_{C'_2} \mathcal{D}_{A'_2}$  to the front of  $\mathcal{D}_{A_1} \partial_{C_1}$  in eq. (50) with our choice of  $n = 2$ , and then transform this supertrace to the following form,

$$(-1)^{|\mathcal{X}||\mathcal{Y}|} \text{STr} \left\{ \partial_{C'_2} \mathcal{D}_{A'_2} \left( \mathcal{D}_{A_1} \partial_{C_1} \frac{1}{\square - m^2} (\square^{-1})^{s_1} \theta^4 \mathcal{F}_{B_1} \partial_{C'_1} \mathcal{D}_{A'_1} \right) \mathcal{D}_{A_2} \partial_{C_2} \frac{1}{\square - m^2} (\square^{-1})^{s_2} \theta^4 \mathcal{F}_{B_2} \right\}, \quad (52)$$

where  $\mathcal{Y} = \partial_{C'_2} \mathcal{D}_{A'_2}$  and  $\mathcal{X}$  represents all factors after  $\partial_{C'_2} \mathcal{D}_{A'_2}$  in eq. (52).

*Step 2:* We define the Leibniz rule as

$$\mathcal{O}\mathcal{G} = (\mathcal{O}\mathcal{G}) + (-1)^{|\mathcal{O}||\mathcal{G}|} \mathcal{G}\mathcal{O}, \quad (53)$$

where  $\mathcal{O}$  is an element of set  $\{\partial_k, D_\alpha, \bar{D}_{\dot{\alpha}}\}$ ,  $\mathcal{G}$  denotes an element of set  $\mathcal{F}$ , or  $\theta^4$ , or  $(\mathcal{D}_A \theta^4)$ , and  $(\mathcal{O}\mathcal{G})$  stands for  $\mathcal{O}$  acting on  $\mathcal{G}$ .

By using the Leibniz rule (eq. (53)) and considering the fact that  $D$  commutes with  $\partial_k$ , we move all  $D$  operators to the front of  $\theta^4 \mathcal{F}_{B_2}$ . At the same time, by using the  $D$  algebraic relations eq. (A2), we can simplify the covariant derivatives to the forms as the elements in set  $\mathcal{D}$  (eq. (46)). Thus, eq. (52) changes to be

$$\text{STr} \left\{ \partial_{C'_2} \partial_{C_1} \frac{1}{\square - m^2} (\square^{-1})^{s_1} (\mathcal{D}_{A'_1} \theta^4) \mathcal{F}_{B'_1} \partial_{C'_1} \partial_{C_2} \frac{1}{\square - m^2} (\square^{-1})^{s_2} \mathcal{D}_{A'_2} \theta^4 \mathcal{F}_{B_2} \right\}, \quad (54)$$

where  $\mathcal{F}_{B'_1}$  is also an element of set  $\mathcal{F}$ , and  $\mathcal{D}_{A'_1}$  and  $\mathcal{D}_{A'_2}$  are also elements of set  $\mathcal{D}$ . Note that  $\mathcal{F}_{B'_1}$ ,  $\mathcal{D}_{A'_1}$ , and  $\mathcal{D}_{A'_2}$  can be determined in terms of the  $D$  algebraic relations eq. (A2) and the factor  $(\mathcal{D}_{A'_1} \theta^4)$  appears in eq. (54).

*Step 3:* The non-vanishing contribution of eq. (54) requires  $\mathcal{D}_{A'_1} = D^2 \bar{D}^2$ . Therefore, eq. (54) reads

$$\text{STr} \left\{ \partial_{C'_2} \partial_{C_1} \frac{1}{\square - m^2} (\square^{-1})^{s_1} \mathcal{F}_{B'_1} \partial_{C'_1} \partial_{C_2} \frac{1}{\square - m^2} (\square^{-1})^{s_2} \mathcal{D}_{A'_2} \theta^4 \mathcal{F}_{B_2} \right\}. \quad (55)$$

*Step 4:* When we calculate the supertrace of the Fermionic coordinates of eq. (55), its non-vanishing contribution requires  $\mathcal{D}_{A'_2} = D^2 \bar{D}^2$ . Thus, eq. (55) reduces to

$$\text{Tr} \left\{ \partial_{C'_2} \partial_{C_1} \frac{1}{\square - m^2} (\square^{-1})^{s_1} \mathcal{F}_{B'_1} \partial_{C'_1} \partial_{C_2} \frac{1}{\square - m^2} (\square^{-1})^{s_2} \theta^4 \mathcal{F}_{B_2} \right\}, \quad (56)$$

where no covariant derivatives  $D$  exist in eq. (56).

*Step 5:* Finally, we compute the momentum integral in eq. (56) by using the dimensional regularization method, and adopt the minimal subtraction scheme which keeps only divergent parts.

The effective actions we obtain are given in Appendix B with our specific method of classification called the 1/2 supersymmetry invariant subsets and bases. For the details of the subsets and bases, see subsections 5.8, 5.9, and 5.10.



## 5.2 Checking of 1/2 Supersymmetry Invariance

We provide in this subsection the method to check the 1/2 supersymmetry invariance of effective actions.

Except for the BFNC parameter factors, the general form of effective actions reads

$$\mathcal{A} = \int d^8z \theta^4 \{ \mathcal{F}_A \mathcal{F}_B \cdots \}, \quad (57)$$

where  $\mathcal{F}_A$  and  $\mathcal{F}_B$  are superfields and elements of set  $\mathcal{F}$  (see eq. (48)), and  $\{ \mathcal{F}_A \mathcal{F}_B \cdots \}$  represents a product of any number of elements in  $\mathcal{F}$ . The supersymmetry transformation obeys the Leibniz rule. We now investigate the supersymmetry transformation of an effective action in which there are two superfields,

$$\delta_\xi \mathcal{A} = \int d^8z \theta^4 \{ (\delta_\xi \mathcal{F}_A) \mathcal{F}_B + \mathcal{F}_A (\delta_\xi \mathcal{F}_B) \}. \quad (58)$$

For example, if we choose  $\mathcal{F}_A = ((\partial)^n D_\alpha \Phi)$ , we have  $\delta_\xi \mathcal{F}_A = ((\partial)^n D_\alpha \delta_\xi \Phi)$ . Considering the transformation of superfield  $\Phi$ ,  $\delta_\xi \Phi = \xi^\beta Q_\beta \Phi$ , where  $\xi^\beta$  is a constant with spinor indices, we then obtain  $\delta_\xi \mathcal{F}_A = ((\partial)^n D_\alpha \xi^\beta Q_\beta \Phi)$ . By using the algebraic relation,  $\{D_\alpha, Q_\beta\} = 0$ , and taking into account the fact that  $\xi^\beta$  is a constant, we move  $Q_\beta$  to the front of  $D_\alpha$  and write  $\delta_\xi \mathcal{F}_A$  as follows,

$$\delta_\xi \mathcal{F}_A = (\xi^\beta (\partial)^n Q_\beta D_\alpha \Phi). \quad (59)$$

Using the identity that  $Q_\beta$  is equal to  $D_\beta$  under the superspace integral  $\int d^8z \theta^4$ ,

$$\int d^8z \theta^4 \{ (\xi^\beta (\partial)^n Q_\beta D_\alpha \Phi) \mathcal{F}_B \} = \int d^8z \theta^4 \{ (\xi^\beta (\partial)^n D_\beta D_\alpha \Phi) \mathcal{F}_B \}, \quad (60)$$

we thus transform covariant derivatives to the simplest form by using the  $D$  algebraic relations. At the end we transform  $\delta_\xi \mathcal{A}$  to the general form of effective actions (see eq. (57)). If the contributions from the two terms in eq. (58) cancel each other,  $\mathcal{A}$  is invariant under 1/2 supersymmetry transformation. In this way, we have checked the effective actions (see our very long final result in Appendix B) and confirmed that our effective actions possess the 1/2 supersymmetry invariance.

## 5.3 New Notations for Presenting Effective Actions

We define the new symbols for presenting effective actions in a concise form,

$$\begin{aligned} \Lambda^{kl} &\equiv \epsilon^{\alpha\beta} \Lambda^k_{\beta} \Lambda^l_{\alpha}, \\ \Lambda^2 &\equiv \eta_{kl} \Lambda^{kl}, \\ \sigma \Lambda \Lambda &\equiv \eta_{kn} \eta_{lo} (\sigma^{kl})^{\alpha\beta} \Lambda^n_{\alpha} \Lambda^o_{\beta}, \end{aligned}$$

$$\begin{aligned}
(\sigma\Lambda^{kl})^{n\alpha} &\equiv (\sigma^{kl})^{\beta\alpha} \Lambda^n{}_{\beta}, \\
(\eta\sigma\Lambda^k)^\alpha &\equiv \eta_{ln} (\sigma^{nk})^{\beta\alpha} \Lambda^l{}_{\beta}, \\
(\eta\sigma\Lambda^k)^l &\equiv \eta_{no} (\sigma^{ok})^{\alpha\beta} \Lambda^n{}_{\alpha} \Lambda^l{}_{\beta}, \\
(\sigma\Lambda\Lambda^{kl})^{no} &\equiv (\sigma^{kl})^{\alpha\beta} \Lambda^n{}_{\alpha} \Lambda^o{}_{\beta}.
\end{aligned} \tag{61}$$

In addition, we hide the superscripts and/or subscripts of Bosonic derivatives, but only show the number of their product, for example,  $\partial_k\partial_l$  is written as  $\partial\partial$ , and also hide the subscripts of Fermionic derivatives, such as  $D$  and  $\bar{D}$  denoting  $D_\alpha$  and  $\bar{D}_{\dot{\beta}}$ , respectively. In particular, for all terms in effective actions we pick out different BFNC parameter factors as a class and different operator factors as the other class, which gives the reader an explicit outline of effective actions. For the details, the reader can confer Appendix B.

As an example, considering the above notations we present  $\mathcal{S}_{\text{NC}}$  (eq. (22)) by separating it into order of  $\Lambda^2$  and order of  $\Lambda^4$  as follows. Note that  $\int d^8z \theta^4$  is omitted.

### 5.3.1 Order of $\Lambda^2$

- There exist five different BFNC parameter factors,

$$\Lambda^{kl}, \quad (\sigma\Lambda\Lambda^{kl})^{no}, \quad \eta^{kl}\Lambda^{no}, \quad \epsilon^{\alpha\beta}\Lambda^{kl}, \quad \epsilon^{\alpha\beta}\epsilon^{\zeta\iota}\Lambda^k{}_{\beta}\Lambda^l{}_{\iota}. \tag{62}$$

- There exist three different operator factors,

$$\partial\partial\Phi(D^2\Phi)(D^2\Phi), \quad \partial\partial(D\Phi)(D\Phi)(D^2\Phi), \quad \partial\partial\partial\partial\Phi^+\Phi^+\Phi^+, \tag{63}$$

where the product of several  $\partial$ 's in the front means it has actions to all the superfields behind it.

### 5.3.2 Order of $\Lambda^4$

- There exists only one BFNC parameter factor,

$$\Lambda^{kl}\Lambda^{no}. \tag{64}$$

- There exists only one operator factor,

$$\partial\partial\partial\partial(D^2\Phi)(D^2\Phi)(D^2\Phi). \tag{65}$$

## 5.4 Formulation and Classification of $\Gamma_{1st}$

As the first step of searching the renormalizable Wess-Zumino model on the BFNC superspace, we calculate the effective action of  $\mathcal{S}_{NC}$  by using the background field method introduced in section 4, and denote it as  $\Gamma_{1st}$ . Incidentally, the order of  $\Lambda^0$  corresponds to the effective action of the Wess-Zumino model on the ordinary superspace and it will not be considered. In the following we give  $\Gamma_{1st}$  in terms of the new notations and the method of classification proposed in subsection 5.3 in order to present the primary effective action in a concise form. We note that in some order of  $\Lambda$  a certain point function does not appear, which means it has no contribution to  $\Gamma_{1st}$ . For instance, at the order of  $\Lambda^2$  the 5- and 6-point functions do not appear, which implies that they have no contributions to  $\Gamma_{1st}$ . As mentioned in subsection 5.2, we verify that  $\Gamma_{1st}$  is invariant under the 1/2 supersymmetry transformation, and we see explicitly that  $\Gamma_{1st}$  cannot be absorbed by  $\mathcal{S}_{NC}$  (eq. (22)) because  $\Gamma_{1st}$  contains many extra terms that do not exist in  $\mathcal{S}_{NC}$ .

### 5.4.1 Order of $\Lambda^2$

- There exist fourteen different BFNC parameter factors,

$$\begin{aligned} & (\eta\sigma\Lambda\Lambda^k)^l, \quad \Lambda^2, \quad \Lambda^{kl}, \quad \epsilon^{\alpha\beta} (\eta\sigma\Lambda\Lambda^k)^l, \quad \epsilon^{\alpha\beta} \Lambda^{kl}, \\ & \Lambda^2 \eta^{kl}, \quad \Lambda^2 \epsilon^{\alpha\beta}, \quad \Lambda^2 \epsilon^{\dot{\alpha}\dot{\beta}}, \quad \Lambda^2 (\bar{\sigma}^k)^{\dot{\alpha}\beta}, \quad \Lambda^2 \eta^{kl} \epsilon^{\alpha\beta}, \\ & \Lambda^{kl} \eta_{ln} (\bar{\sigma}^n)^{\dot{\alpha}\beta}, \quad \eta_{kl} (\bar{\sigma}^l)^{\dot{\alpha}\beta} (\eta\sigma\Lambda\Lambda^k)^n, \quad \epsilon^{\alpha\beta} \epsilon^{\zeta\iota} \Lambda^k_{\beta} \Lambda^l_{\iota}, \quad \epsilon^{\alpha\beta} \eta_{kl} (\sigma\Lambda^{ln})^{\alpha\zeta} \Lambda^k_{\beta}. \end{aligned} \quad (66)$$

- There exist three different operator factors,

1. The 2-point function contains four different forms,

$$\partial\partial\Phi(D^2\Phi), \quad \partial\partial(D\Phi)(D\Phi), \quad \partial\partial(D^2\Phi)\Phi^+, \quad \partial\partial\partial\partial(D^2\Phi)\Phi^+; \quad (67)$$

2. The 3-point function contains five different forms,

$$\begin{aligned} & (D^2\Phi)(D^2\Phi)(\bar{D}^2\Phi^+), \quad \partial(D\Phi)(D^2\Phi)(\bar{D}\Phi^+), \quad \partial\partial\Phi(D^2\Phi)\Phi^+, \\ & \partial\partial(D\Phi)(D\Phi)\Phi^+, \quad \partial\partial(D^2\Phi)\Phi^+\Phi^+; \end{aligned} \quad (68)$$

3. The 4-point function contains five different forms,

$$\begin{aligned} & (D^2\Phi)(D^2\Phi)(\bar{D}\Phi^+)(\bar{D}\Phi^+), \quad (D^2\Phi)(D^2\Phi)\Phi^+(\bar{D}^2\Phi^+), \\ & \partial(D\Phi)(D^2\Phi)\Phi^+(\bar{D}\Phi^+), \quad \partial\partial\Phi(D^2\Phi)\Phi^+\Phi^+, \quad \partial\partial(D\Phi)(D\Phi)\Phi^+\Phi^+. \end{aligned} \quad (69)$$

### 5.4.2 Order of $\Lambda^4$

- There exist seven different BFNC parameter factors,

$$\begin{aligned} \Lambda^2 \Lambda^{kl}, \quad \Lambda^2 (\sigma \Lambda \Lambda^{kl})^{no}, \quad \Lambda^{kl} (\eta \sigma \Lambda \Lambda^n)^o, \quad \Lambda^{kl} \Lambda^{no}, \quad \Lambda^2 \eta^{kl} \Lambda^{no}, \\ \Lambda^2 \epsilon^{\alpha\beta} \Lambda^{kl}, \quad \Lambda^{kl} \eta_{ln} (\sigma \Lambda \Lambda^{no})^{pq}. \end{aligned} \quad (70)$$

- There exist five different operator factors,

1. The 2-point function contains two different forms,

$$\partial \partial (D^2 \Phi) (D^2 \Phi), \quad \partial \partial \partial \partial (D^2 \Phi) (D^2 \Phi); \quad (71)$$

2. The 3-point function contains four different forms,

$$\begin{aligned} \partial \partial \Phi (D^2 \Phi) (D^2 \Phi), \quad \partial \partial (D \Phi) (D \Phi) (D^2 \Phi), \quad \partial \partial (D^2 \Phi) (D^2 \Phi) \Phi^+, \\ \partial \partial \partial \partial (D^2 \Phi) (D^2 \Phi) \Phi^+; \end{aligned} \quad (72)$$

3. The 4-point function contains four different forms,

$$\begin{aligned} \partial \partial \Phi (D^2 \Phi) (D^2 \Phi) \Phi^+, \quad \partial \partial (D \Phi) (D \Phi) (D^2 \Phi) \Phi^+, \quad \partial \partial (D^2 \Phi) (D^2 \Phi) \Phi^+ \Phi^+, \\ \partial \partial \partial \partial (D^2 \Phi) (D^2 \Phi) \Phi^+ \Phi^+; \end{aligned} \quad (73)$$

4. The 5-point function contains three different forms,

$$\begin{aligned} \partial \partial \Phi (D^2 \Phi) (D^2 \Phi) \Phi^+ \Phi^+, \quad \partial \partial (D \Phi) (D \Phi) (D^2 \Phi) \Phi^+ \Phi^+, \\ \partial \partial (D^2 \Phi) (D^2 \Phi) \Phi^+ \Phi^+ \Phi^+; \end{aligned} \quad (74)$$

5. The 6-point function contains two different forms,

$$\partial \partial \Phi (D^2 \Phi) (D^2 \Phi) \Phi^+ \Phi^+ \Phi^+, \quad \partial \partial (D \Phi) (D \Phi) (D^2 \Phi) \Phi^+ \Phi^+ \Phi^+. \quad (75)$$

### 5.4.3 Order of $\Lambda^6$

- There exists only one BFNC parameter factor,

$$\Lambda^2 \Lambda^{kl} \Lambda^{no}. \quad (76)$$

- There exist three different operator factors,

1. The 4-point function contains only one form,

$$\partial\partial\partial\partial (D^2\Phi) (D^2\Phi) (D^2\Phi) \Phi^+; \quad (77)$$

2. The 5-point function contains only one form,

$$\partial\partial\partial\partial (D^2\Phi) (D^2\Phi) (D^2\Phi) \Phi^+ \Phi^+; \quad (78)$$

3. The 6-point function contains only one form,

$$\partial\partial\partial\partial (D^2\Phi) (D^2\Phi) (D^2\Phi) \Phi^+ \Phi^+ \Phi^+. \quad (79)$$

## 5.5 Restriction to Order of $\Lambda^2$

Before continuing our analysis, let us temporally recall the corresponding situation for the NAC case [11] where the effective action of the Wess-Zumino model contains only one term,

$$\int d^8z U (D^2\Phi)^2, \quad U = \theta^2 \bar{\theta}^2 C^2, \quad (80)$$

where  $C$  is the NAC parameter. In that case the renormalizable action can be obtained by adding the effective action to the deformed Wess-Zumino model on the NAC superspace.

We turn to our case on the BFNC superspace, where the primary effective action  $\Gamma_{1\text{st}}$  cannot be absorbed by the deformed action (eq. (22)). In our case the number of terms that should be added to  $\mathcal{S}_{\text{NC}}$  (eq. (22)) is very large, for instance,  $\Gamma_{1\text{st}}$  contains 68 terms only at the order of  $\Lambda^2$ . So, it is a tremendously exciting challenge to find out the successive effective actions needed.

In general, if a model can be modified to be renormalizable, it must be renormalizable at each order of  $\Lambda$ . On the BFNC superspace the possible orders of  $\Lambda$  are even, that is,  $\Lambda^2$ ,  $\Lambda^4$ ,  $\Lambda^6$ , and  $\Lambda^8$ , and higher orders than  $\Lambda^8$  must be vanishing due to the Fermionic number of the BFNC parameters, where the lowest order of approximation is  $\Lambda^2$ . In order to obtain the basic information of the renormalization property of the Wess-Zumino model on the BFNC superspace, we restrict our analysis only at the order of  $\Lambda^2$  in the following calculations of the successive effective actions, such as  $\Gamma_{2\text{nd}}$ ,  $\Gamma_{3\text{rd}}$ , and  $\Gamma_{4\text{th}}$ .

## 5.6 Formulation and Classification of $\Gamma_{2\text{nd}}$

### 5.6.1 Method

We start from  $\mathcal{S}_{\text{NC}} + \Gamma_{1\text{st}}$  and calculate its effective action called the secondary effective action  $\Gamma_{2\text{nd}}$  up to the order of  $\Lambda^2$ . Note that we remove the divergent coefficient  $\frac{1}{\epsilon}$  from  $\Gamma_{1\text{st}}$  and still use the same symbol to denote the primary effective action. The detailed procedure is as follows.

In eq. (22),  $\mathcal{S}_{\text{NC}}$  is written as the sum of the ordinary part  $\mathcal{S}_{\text{WZ}}$  and the noncommutative part  $\mathcal{S}_{\Lambda}$ ,  $\mathcal{S}_{\text{NC}} = \mathcal{S}_{\text{WZ}} + \mathcal{S}_{\Lambda}$ . Here we define

$$\mathcal{S}_{(1)} \equiv \mathcal{S}_{\text{WZ}} + \mathcal{S}_{\Lambda}(\Lambda^2) + \Gamma_{1\text{st}}(\Lambda^2), \quad (81)$$

where  $\mathcal{S}_{\Lambda}(\Lambda^2)$  stands for the  $\Lambda^2$  part of  $\mathcal{S}_{\Lambda}$ , and  $\Gamma_{1\text{st}}(\Lambda^2)$  for the  $\Lambda^2$  part of  $\Gamma_{1\text{st}}$ .  $\mathcal{S}_{\Lambda}(\Lambda^2)$  is given in subsection 5.3.1 in a concise form or in eq. (22) in detail, and  $\Gamma_{1\text{st}}(\Lambda^2)$  is also known, see subsection 5.4.1 in a concise form or Appendix B in detail. Our goal is  $\Gamma_{2\text{nd}}(\Lambda^2)$  which is the effective action of  $\mathcal{S}_{(1)}$ .

In fact, the effective action contributed by the vertices of  $\mathcal{S}_{\Lambda}(\Lambda^2)$  is just  $\Gamma_{1\text{st}}(\Lambda^2)$  which is already known, so we only need to compute the effective action of  $\mathcal{S}_{\text{WZ}} + \Gamma_{1\text{st}}(\Lambda^2)$ .

By following the general procedure of the background field method, we divide  $\mathcal{S}_{\text{WZ}}$  (eq. (14)) into the free and interacting parts,  $\mathcal{S}_{\text{WZ}} = \mathcal{S}_0 + \mathcal{S}_{\text{int}}$ , and still use  $\mathcal{S}_{\text{GF}}$  (see eq. (28)) as the gauge fixing term.

As in calculating  $\Gamma_{1\text{st}}$ , matrix  $M$  corresponding to  $\Gamma_{2\text{nd}}$  is still determined by  $\mathcal{S}_0 + \mathcal{S}_{\text{GF}}$ . However, different from the case of  $\Gamma_{1\text{st}}$ , a new matrix  $V_{(1)}$  related to the interacting part for  $\Gamma_{2\text{nd}}$  is determined by  $\mathcal{S}_{\text{int}} + \Gamma_{1\text{st}}(\Lambda^2)$ . In addition, we put the 2-point function of  $\Gamma_{1\text{st}}(\Lambda^2)$  into  $V_{(1)}$  rather than into  $M$  in order to avoid  $M$  being not invertible. With all these considerations, we can obtain  $\Gamma_{2\text{nd}}(\Lambda^2)$ .

### 5.6.2 Result

By comparing the BFNC parameter factors and operator ones of  $\Gamma_{2\text{nd}}(\Lambda^2)$  with that of  $\mathcal{S}_{(1)}$ , we find some new forms that do not appear in  $\mathcal{S}_{(1)}$ . We list them as follows.

- There exist thirteen different BFNC parameter factors,

$$\begin{aligned} & \sigma\Lambda\Lambda, \quad \Lambda^2(\sigma^{kl})^{\alpha\beta}, \quad \sigma\Lambda\Lambda\eta^{kl}, \quad \sigma\Lambda\Lambda\epsilon^{\alpha\beta}, \quad \sigma\Lambda\Lambda\epsilon^{\dot{\alpha}\dot{\beta}}, \quad \sigma\Lambda\Lambda(\bar{\sigma}^k)^{\dot{\alpha}\beta}, \\ & \epsilon^{\alpha\beta}(\eta\sigma\Lambda^k)^{\zeta}\Lambda^l_{\beta}, \quad \Lambda^2\eta^{kl}\eta^{no}, \quad \Lambda^{kl}\eta_{ln}(\sigma^{no})^{\alpha\beta}, \quad \sigma\Lambda\Lambda\eta^{kl}\epsilon^{\alpha\beta}, \\ & \eta_{kl}(\bar{\sigma}^l)^{\dot{\alpha}\beta}(\eta\sigma\Lambda^n)^k, \quad \epsilon^{\alpha\beta}\eta_{kl}(\eta\sigma\Lambda^l)^{\zeta}\Lambda^k_{\beta}, \quad \epsilon^{\alpha\beta}\epsilon^{\zeta\iota}\epsilon^{klno}\eta_{np}\eta_{oq}\Lambda^p_{\beta}\Lambda^q_{\iota}. \end{aligned} \quad (82)$$

- There exist six different operator factors,

1. The 1-point function contains only one form,

$$D^2\Phi; \quad (83)$$

2. The 2-point function contains ten different forms,

$$\begin{aligned} & \Phi(D^2\Phi), \quad (D\Phi)(D\Phi), \quad (D^2\Phi)(D^2\Phi), \quad (D^2\Phi)(\bar{D}^2\Phi^+), \\ & (D^2\Phi)\Phi^+, \quad \partial(D\Phi)(\bar{D}\Phi^+), \quad \partial\partial\Phi\Phi^+, \quad \partial\partial(D^2\Phi)(D^2\Phi), \\ & \partial\partial\Phi^+\Phi^+, \quad \partial\partial\partial\partial\Phi^+\Phi^+; \end{aligned} \quad (84)$$

3. The 3-point function contains thirteen different forms,

$$\begin{aligned} & \Phi\Phi(D^2\Phi), \quad \Phi(D\Phi)(D\Phi), \quad \Phi(D^2\Phi)(D^2\Phi), \quad \Phi(D^2\Phi)\Phi^+, \\ & (D\Phi)(D\Phi)(D^2\Phi), \quad (D\Phi)(D\Phi)\Phi^+, \quad (D^2\Phi)(D^2\Phi)\Phi^+, \\ & (D^2\Phi)(\bar{D}\Phi^+)(\bar{D}\Phi^+), \quad (D^2\Phi)\Phi^+(\bar{D}^2\Phi^+), \quad (D^2\Phi)\Phi^+\Phi^+, \\ & \partial(D\Phi)\Phi^+(\bar{D}\Phi^+), \quad \partial\partial\Phi\Phi^+\Phi^+, \quad \partial\partial\Phi^+\Phi^+\Phi^+; \end{aligned} \quad (85)$$

4. The 4-point function contains fourteen different forms,

$$\begin{aligned} & \Phi\Phi(D^2\Phi)(D^2\Phi), \quad \Phi\Phi(D^2\Phi)\Phi^+, \quad \Phi(D\Phi)(D\Phi)(D^2\Phi), \\ & \Phi(D\Phi)(D\Phi)\Phi^+, \quad \Phi(D^2\Phi)(D^2\Phi)\Phi^+, \quad \Phi(D^2\Phi)\Phi^+\Phi^+, \\ & (D\Phi)(D\Phi)(D^2\Phi)\Phi^+, \quad (D\Phi)(D\Phi)\Phi^+\Phi^+, \quad (D^2\Phi)\Phi^+(\bar{D}\Phi^+)(\bar{D}\Phi^+), \\ & (D^2\Phi)\Phi^+\Phi^+(\bar{D}^2\Phi^+), \quad (D^2\Phi)\Phi^+\Phi^+\Phi^+, \quad \partial(D\Phi)\Phi^+\Phi^+(\bar{D}\Phi^+), \\ & \partial\partial\Phi\Phi^+\Phi^+\Phi^+, \quad \partial\partial\Phi^+\Phi^+\Phi^+\Phi^+; \end{aligned} \quad (86)$$

5. The 5-point function contains ten different forms,

$$\begin{aligned} & \Phi\Phi(D^2\Phi)(D^2\Phi)\Phi^+, \quad \Phi\Phi(D^2\Phi)\Phi^+\Phi^+, \quad \Phi(D\Phi)(D\Phi)(D^2\Phi)\Phi^+, \\ & \Phi(D\Phi)(D\Phi)\Phi^+\Phi^+, \quad \Phi(D^2\Phi)\Phi^+\Phi^+\Phi^+, \quad (D\Phi)(D\Phi)\Phi^+\Phi^+\Phi^+, \\ & (D^2\Phi)\Phi^+\Phi^+(\bar{D}\Phi^+)(\bar{D}\Phi^+), \quad (D^2\Phi)\Phi^+\Phi^+\Phi^+(\bar{D}^2\Phi^+), \\ & \partial(D\Phi)\Phi^+\Phi^+\Phi^+(\bar{D}\Phi^+), \quad \partial\partial\Phi\Phi^+\Phi^+\Phi^+\Phi^+; \end{aligned} \quad (87)$$

6. The 6-point function contains two different forms,

$$\Phi\Phi(D^2\Phi)\Phi^+\Phi^+\Phi^+, \quad \Phi(D\Phi)(D\Phi)\Phi^+\Phi^+\Phi^+. \quad (88)$$

Because of the existence of the above new forms,  $\Gamma_{\text{2nd}}(\Lambda^2)$  cannot be absorbed by  $\mathcal{S}_{(1)}$ , which means  $\mathcal{S}_{(1)}$  is still not renormalizable. Different from ref. [11], we have to compute the next successive effective action  $\Gamma_{\text{3rd}}$ .

## 5.7 Formulation and Classification of $\Gamma_{3\text{rd}}$

### 5.7.1 Method

Similar to the calculation of  $\Gamma_{2\text{nd}}(\Lambda^2)$ , we define

$$\mathcal{S}_{(2)} \equiv \mathcal{S}_{(1)} + \Gamma_{2\text{nd}}(\Lambda^2), \quad (89)$$

and calculate the effective action of  $\mathcal{S}_{(2)}$ . Because the effective action contributed by the vertices of  $\mathcal{S}_\Lambda(\Lambda^2) + \Gamma_{1\text{st}}(\Lambda^2)$  is already known, what we need to do is just to calculate the effective action of  $\mathcal{S}_{\text{WZ}} + \Gamma_{2\text{nd}}(\Lambda^2)$ .

Different from the calculation of  $\Gamma_{2\text{nd}}(\Lambda^2)$ ,  $V_{(2)}$  contains the 2-point function of  $\Gamma_{2\text{nd}}(\Lambda^2)$ , and it is determined by  $\mathcal{S}_{\text{int}} + \Gamma_{2\text{nd}}(\Lambda^2)$ . Exactly following the way described in subsection 5.6.1, we get the desired result  $\Gamma_{3\text{rd}}(\Lambda^2)$ .

### 5.7.2 Result

By comparing the BFNC parameter factors and operator ones of  $\Gamma_{3\text{rd}}(\Lambda^2)$  with that of  $\mathcal{S}_{(2)}$ , we find the following new BFNC parameter factors that do not appear in  $\mathcal{S}_{(2)}$ ,

$$\eta^{kl}(\eta\sigma\Lambda\Lambda^n)^o, \quad \sigma\Lambda\Lambda(\sigma^{kl})^{\alpha\beta}, \quad (90)$$

but the operator factors in  $\Gamma_{3\text{rd}}(\Lambda^2)$  are exactly same as that in  $\mathcal{S}_{(2)}$ . Therefore, there are still some terms in  $\Gamma_{3\text{rd}}(\Lambda^2)$  that cannot be absorbed by  $\mathcal{S}_{(2)}$ . The reason is that these terms originate from the combination of BFNC parameter and operator factors. This implies that we have to continue the iterative procedure in order to find the renormalizable Wess-Zumino model that possesses the 1/2 supersymmetry invariance on the BFNC superspace. At this stage, we notice that many terms in  $\Gamma_{1\text{st}}(\Lambda^2)$ ,  $\Gamma_{2\text{nd}}(\Lambda^2)$ , and  $\Gamma_{3\text{rd}}(\Lambda^2)$  are same, which brings us to compare the three effective actions and to analyze their structures.

## 5.8 1/2 Supersymmetry Invariant Subset

Based on the Wess-Zumino action (eq. (13)) and the definition of  $\star$ -product (eq. (17)),  $\mathcal{S}_{\text{NC}}$  (eq. (22)) is invariant under the 1/2 supersymmetry transformation (see eq. (24)). By using the background field method, we can ensure that all of the effective actions are invariant under the 1/2 supersymmetry transformation. In  $\mathcal{S}_{\text{NC}}$ , the noncommutative part  $\mathcal{S}_\Lambda$  contains the contributions from the orders of  $\Lambda^2$  and  $\Lambda^4$ , see subsections 5.3.1 and 5.3.2, and each order contribution as a close set is invariant under the



1/2 supersymmetry transformation. In the present work, we focus on the order of  $\Lambda^2$  and further find that  $\mathcal{S}_\Lambda(\Lambda^2)$  can be separated into several classes, each of which as a close set contains the minimal number of terms and is invariant under the 1/2 supersymmetry transformation. Any two classes have no common terms. So we introduce the concept of 1/2 supersymmetry invariant subsets and name every class as a 1/2 supersymmetry invariant subset.

The number of terms in  $\mathcal{S}_\Lambda(\Lambda^2)$  is small, so it is easy to find its subsets. However, each of the effective actions  $\Gamma_{1\text{st}}(\Lambda^2)$ ,  $\Gamma_{2\text{nd}}(\Lambda^2)$ , and  $\Gamma_{3\text{rd}}(\Lambda^2)$  contains a quite large number of terms, and thus a systematic method is needed to find the 1/2 supersymmetry invariant subsets for each effective action. We explain our proposal as follows by using  $\Gamma_{1\text{st}}(\Lambda^2)$  as an example.

*Step 1:* After transforming  $\Gamma_{1\text{st}}(\Lambda^2)$  to its simplest form, we still have 68 terms which are labelled arbitrarily and denoted as  $L_i$ ,

$$\Gamma_{1\text{st}}(\Lambda^2) = \sum_{i=1}^n L_i, \quad (91)$$

where  $n = 68$ .

*Step 2:* We multiply  $L_i$  by a constant  $X_i$  and sum them, and denote the result as  $A$ ,

$$A = \sum_{i=1}^n X_i L_i. \quad (92)$$

*Step 3:* Making the 1/2 supersymmetry transformation to  $A$  and simplifying  $\delta_\xi A$ , we obtain that it consists of  $n'$  terms,

$$\delta_\xi A = \sum_{j=1}^{n'} Y_j L'_j, \quad (93)$$

where  $L'_j$  is different from  $L_i$ . Thus we derive that  $Y_j$  is a linear function of  $X_i$ 's,

$$Y_j = \sum_{i=1}^n c_{ji} X_i, \quad (94)$$

where  $c_{ji}$  is real. For every  $j$ ,  $c_{ji} = 0$  for some  $i$ 's, that is, those  $X_i$ 's do not appear in  $Y_j$ .

For each  $j$ ,  $j = 1, \dots, n'$ , we construct set  $U_j$  using the parameters  $X_i$ 's in  $Y_j$ ,

$$U_j = \{X_{i_1}, X_{i_2}, \dots\}, \quad (95)$$

where  $\{i_1, i_2, \dots\}$  is a subset of  $\{1, \dots, n\}$ . Then we obtain set  $W$  which contains  $n'$  elements,

$$W = \{U_1, U_2, \dots, U_{n'}\}. \quad (96)$$

*Step 4:* For every pair of  $(j, j')$ , where  $j \neq j'$ , if  $U_j \cap U_{j'} \neq \emptyset$ , we define

$$U_{jj'} \equiv U_j \cup U_{j'}, \quad (97)$$

and remove  $U_j$  and  $U_{j'}$  from  $W$ , but add  $U_{jj'}$  to  $W$ .

Repeating the above process, we obtain at the end,

$$W' = \{I_1, I_2, \dots, I_m, \dots\}, \quad (98)$$

where its element

$$I_m = \{X_{m_1}, X_{m_2}, \dots\} \quad (99)$$

satisfies  $I_m \cap I_{m'} = \emptyset$  when  $m \neq m'$ , and  $\{m_1, m_2, \dots\}$  is a subset of  $\{1, \dots, n\}$ .

*Step 5:* For set  $I_m = \{X_{m_1}, X_{m_2}, \dots\}$ , we sum the elements of its corresponding set  $\{L_{m_1}, L_{m_2}, \dots\}$ , and denote it as  $f_m$ ,

$$f_m = \sum_{i=\{m_1, m_2, \dots\}} L_i. \quad (100)$$

Thus,  $f_m$  provides one 1/2 supersymmetry invariant subset. We can deduce that there exist 17 invariant subsets in  $\Gamma_{1st}(\Lambda^2)$ .

By using the above proposal with five steps, we can separate the other three actions,  $\mathcal{S}_\Lambda(\Lambda^2)$ ,  $\Gamma_{2nd}(\Lambda^2)$ , and  $\Gamma_{3rd}(\Lambda^2)$ , into their 1/2 supersymmetry invariant subsets whose numbers are 4, 64, and 73, respectively. By comparing the invariant subsets of the four actions, we see that some subsets have same BFNC parameter and operator factors but different coefficients. After summing up all of the subsets of the four actions, we finally determine that there are 74 independent subsets in total, each of which is invariant under the 1/2 supersymmetry transformation, see Appendix B.1 for the details where the explanation of four parameters  $a_0$ ,  $a_1$ ,  $a_2$ , and  $a_3$  is provided below.

## 5.9 Analysis of Effective Actions by Invariant Subsets

As mentioned in the above that some terms have same BFNC parameter and operator factors but different coefficients in the actions  $\mathcal{S}_\Lambda(\Lambda^2)$ ,  $\Gamma_{1st}(\Lambda^2)$ ,  $\Gamma_{2nd}(\Lambda^2)$ , and  $\Gamma_{3rd}(\Lambda^2)$ , we present these actions in an explicit way related to invariant subsets, that is, we show the ingredients of an invariant subset provided by the four actions.

We construct a new action,

$$\Gamma(\Lambda^2) = a_0 \mathcal{S}_\Lambda(\Lambda^2) + a_1 \Gamma_{1st}(\Lambda^2) + a_2 \Gamma_{2nd}(\Lambda^2) + a_3 \Gamma_{3rd}(\Lambda^2), \quad (101)$$

where  $a_0$ ,  $a_1$ ,  $a_2$ , and  $a_3$  are parameters which show intersections of invariant subsets of different actions. All invariant subsets for  $\Gamma(\Lambda^2)$  are given in Appendix B.1 and denoted by  $f_i$ , where  $i = 1, \dots, 74$ . Thus,  $\Gamma(\Lambda^2)$  can be rewritten as

$$\Gamma(\Lambda^2) = \sum_{i=1}^{74} f_i. \quad (102)$$

In order to explicitly show the intersections of invariant subsets of different actions, we take the invariant subset No. 29 (see Appendix B.1),  $f_{29}$ , as an example,

$$\begin{aligned}
f_{29} = & \frac{5(-18g^5a_2 + 23g^7a_3)}{13824} \epsilon^{\alpha\beta} \Lambda^{kl} \theta^4 (D_\beta \Phi) \partial_l \partial_k (D_\alpha \Phi) (D^2 \Phi) \\
& + \frac{-192ga_0 - 36g^5a_2 + 55g^7a_3}{3072} \epsilon^{\alpha\beta} \epsilon^{\zeta\iota} \Lambda^k_{\beta} \Lambda^l_{\iota} \theta^4 \partial_k (D_\alpha \Phi) \partial_l (D_\zeta \Phi) (D^2 \Phi) \\
& + \frac{-144ga_0 - 45g^5a_2 + 68g^7a_3}{4608} \Lambda^{kl} \theta^4 \Phi \partial_k (D^2 \Phi) \partial_l (D^2 \Phi) \\
& + \frac{-864ga_0 - 360g^5a_2 + 523g^7a_3}{27648} \Lambda^{kl} \theta^4 \Phi (D^2 \Phi) \partial_l \partial_k (D^2 \Phi) \\
& + \frac{-3456ga_0 - 504g^5a_2 + 851g^7a_3}{55296} \epsilon^{\alpha\beta} \Lambda^{kl} \theta^4 \partial_k (D_\alpha \Phi) \partial_l (D_\beta \Phi) (D^2 \Phi). \tag{103}
\end{aligned}$$

The first line means that the combination of BFNC parameter and operator factors,

$$\epsilon^{\alpha\beta} \Lambda^{kl} \theta^4 (D_\beta \Phi) \partial_l \partial_k (D_\alpha \Phi) (D^2 \Phi),$$

comes from both  $\Gamma_{2\text{nd}}(\Lambda^2)$  and  $\Gamma_{3\text{rd}}(\Lambda^2)$ , but the two effective actions provide different coefficients which are  $\frac{-90g^5}{13824}$  and  $\frac{115g^7}{13824}$ , respectively. The other four lines have the similar meaning, and emerge from  $\mathcal{S}_\Lambda(\Lambda^2)$ ,  $\Gamma_{2\text{nd}}(\Lambda^2)$ , and  $\Gamma_{3\text{rd}}(\Lambda^2)$  because  $a_0$ ,  $a_2$ , and  $a_3$  appear. Moreover,  $\Gamma_{1\text{st}}(\Lambda^2)$  has no contribution to  $f_{29}$  as  $a_1$  does not appear. In this way, one can clearly see the ingredients of every invariant subset provided by  $\mathcal{S}_\Lambda(\Lambda^2)$ ,  $\Gamma_{1\text{st}}(\Lambda^2)$ ,  $\Gamma_{2\text{nd}}(\Lambda^2)$ , and  $\Gamma_{3\text{rd}}(\Lambda^2)$ .

## 5.10 Basis of Supersymmetry Invariant Subset

Based on the analysis made to the 74 supersymmetry invariant subsets, we try to construct more general 1/2 supersymmetry invariant subsets in order to deduce the one-loop renormalizable Wess-Zumino action on the BFNC superspace. For the sake of concreteness, we take  $f_{29}$  as an example for proposing our idea.  $f_{29}$  has five different combinations of BFNC parameter and operator factors, in each combination the coefficients from  $\mathcal{S}_\Lambda(\Lambda^2)$ ,  $\Gamma_{2\text{nd}}(\Lambda^2)$ , and  $\Gamma_{3\text{rd}}(\Lambda^2)$  are different. Four terms corresponding to  $a_0$  are from  $\mathcal{S}_\Lambda(\Lambda^2)$ , five terms corresponding to  $a_2$  are from  $\Gamma_{2\text{nd}}(\Lambda^2)$ , and the last five terms corresponding to  $a_3$  are from  $\Gamma_{3\text{rd}}(\Lambda^2)$ . We have already known that the three groups of terms possess the 1/2 supersymmetry invariance because they belong to the invariant subsets of  $\mathcal{S}_\Lambda(\Lambda^2)$ ,  $\Gamma_{2\text{nd}}(\Lambda^2)$ , and  $\Gamma_{3\text{rd}}(\Lambda^2)$ , respectively. To construct a more general 1/2 supersymmetry invariant subset than  $f_{29}$ , we try to multiply every combination by an arbitrary parameter, then look for the constraints of the parameters by demanding that the generalization of  $f_{29}$  is 1/2 supersymmetry invariant, and at last give the generalization that contains arbitrary parameters (other than  $a_0$ ,  $a_2$ , and  $a_3$ ). In this way, we introduce the concept of a basis for every 1/2 supersymmetry invariant subset, meaning a

generalized invariant subset with arbitrary parameters. We describe our proposal in more details below on how to work out a basis of a 1/2 supersymmetry invariant subset.

We write an invariant subset  $f_i$  as

$$f_i = \sum_{j=1}^{n_i} C_{ij} L_j, \quad (104)$$

where  $C_{ij}$  stands for a coefficient and  $L_j$  a combination of BFNC parameter and operator factors,  $i = 1, 2, \dots, 74$  the total number of invariant subsets, and  $n_i$  means the number of terms in  $f_i$ .

*Step 1:* By introducing three groups of parameters  $x_{i,j}$ ,  $y_{i,j}$ , and  $z_{i,j}$ , we construct a basis  $A_i$ ,

$$A_i \equiv \sum_{j=1}^{n_i} (x_{i,j} + y_{i,j} \Lambda^2 + z_{i,j} \sigma \Lambda \Lambda) L_j. \quad (105)$$

*Step 2:* Remove from  $A_i$  those terms where the power of  $\Lambda$  is greater than 2 because  $L_j$  contains  $\Lambda^2$  and  $\sigma \Lambda \Lambda$  in its BFNC parameter factors. Then some components of  $x_{i,j}$ ,  $y_{i,j}$ , and  $z_{i,j}$  can be eliminated. We denote the result as  $A'_i$ .

*Step 3:* Making the 1/2 supersymmetry transformation for  $A'_i$ , we obtain

$$\delta A'_i = \sum_{k=1}^{n'_i} G_{i,k}(x, y, z) L'_k, \quad (106)$$

where  $G_{i,k}(x, y, z)$  is a linear function of  $x_{i,j}$ ,  $y_{i,j}$ , and  $z_{i,j}$ , where  $j = 1, \dots, n_i$ .

*Step 4:* Let  $A'_i$  be a 1/2 invariant subset, we have  $\delta A'_i = 0$ . Because  $L'_k$ 's are independent to each other, we get

$$G_{i,k}(x, y, z) = 0, \quad (107)$$

which is a set of constraints on parameters  $x_{i,j}$ ,  $y_{i,j}$ , and  $z_{i,j}$ .

*Step 5:* Not all of  $x_{i,j}$ ,  $y_{i,j}$ , and  $z_{i,j}$  are independent. We eliminate the dependent parameters from  $A'_i$  and then obtain a basis.

As an example, we explain the basis  $B_{29}$  that corresponds to the subset  $f_{29}$  (eq. (103)). By using the above proposal, we obtain

$$\begin{aligned} B_{29} = & (-2x_{29,3} + 2x_{29,4}) \epsilon^{\alpha\beta} \Lambda^{kl} \theta^4 (D_\beta \Phi) \partial_l \partial_k (D_\alpha \Phi) (D^2 \Phi) \\ & + (-6x_{29,3} + 4x_{29,4} + 2x_{29,5}) \epsilon^{\alpha\beta} \epsilon^{\zeta\iota} \Lambda^k {}_\beta \Lambda^l {}_\iota \theta^4 \partial_k (D_\alpha \Phi) \partial_l (D_\zeta \Phi) (D^2 \Phi) \\ & + x_{29,3} \Lambda^{kl} \theta^4 \Phi \partial_k (D^2 \Phi) \partial_l (D^2 \Phi) \\ & + x_{29,4} \Lambda^{kl} \theta^4 \Phi (D^2 \Phi) \partial_l \partial_k (D^2 \Phi) \\ & + x_{29,5} \epsilon^{\alpha\beta} \Lambda^{kl} \theta^4 \partial_k (D_\alpha \Phi) \partial_l (D_\beta \Phi) (D^2 \Phi), \end{aligned} \quad (108)$$

where  $x_{29,3}$ ,  $x_{29,4}$ , and  $x_{29,5}$  are independent parameters that are related to lines 3, 4, and 5 of  $f_{29}$ . The parameters  $x_{29,1}$  and  $x_{29,2}$  related to lines 1 and 2 of  $f_{29}$  are not independent but determined by the three independent parameters, i.e.,  $x_{29,1} = -2x_{29,3} + 2x_{29,4}$  and  $x_{29,2} = -6x_{29,3} + 4x_{29,4} + 2x_{29,5}$ .

We observe that in subset  $f_{29}$  the terms that belong to actions  $\mathcal{S}_\Lambda(\Lambda^2)$ ,  $\Gamma_{2\text{nd}}(\Lambda^2)$ , and  $\Gamma_{3\text{rd}}(\Lambda^2)$ , respectively, can be obtained by choosing special values for the independent parameters  $x_{29,3}$ ,  $x_{29,4}$ , and  $x_{29,5}$  in  $B_{29}$  (eq. (108)). We list the corresponding values for  $\mathcal{S}_\Lambda(\Lambda^2)$ ,  $\Gamma_{2\text{nd}}(\Lambda^2)$ , or  $\Gamma_{3\text{rd}}(\Lambda^2)$  in the following three lines, respectively,

$$\begin{aligned} x_{29,3} &\rightarrow -\frac{1}{32}, & x_{29,4} &\rightarrow -\frac{1}{32}, & x_{29,5} &\rightarrow -\frac{1}{16}; \\ x_{29,3} &\rightarrow -\frac{5}{512}, & x_{29,4} &\rightarrow -\frac{5}{384}, & x_{29,5} &\rightarrow -\frac{7}{768}; \\ x_{29,3} &\rightarrow \frac{17}{1152}, & x_{29,4} &\rightarrow \frac{523}{27648}, & x_{29,5} &\rightarrow \frac{851}{55296}. \end{aligned} \quad (109)$$

This means if we replace the parameters  $x_{29,3}$ ,  $x_{29,4}$ , and  $x_{29,5}$  in  $B_{29}$  by the values of the first line of eq. (109), we obtain the contribution from  $\mathcal{S}_\Lambda(\Lambda^2)$  to the subset  $f_{29}$ , which is equivalently to set  $a_0 = 1$ ,  $a_2 = 0$ , and  $a_3 = 0$  in  $f_{29}$  (eq. (103)).

We deal with every subset  $f_i$  using our proposal, and obtain its corresponding basis  $B_i$ , where  $i = 1, \dots, 74$ . That is, there are 74 bases in total that are given in Appendix B.2. By using the bases, we shall get the desired renormalizable action.

## 5.11 Renormalizable Action

According to the background field method, we need to calculate the effective action  $\Gamma_{4\text{th}}(\Lambda^2)$  based on  $\mathcal{S}_{\text{WZ}} + \mathcal{S}_\Lambda(\Lambda^2) + \Gamma_{1\text{st}}(\Lambda^2) + \Gamma_{2\text{nd}}(\Lambda^2) + \Gamma_{3\text{rd}}(\Lambda^2)$ . From the above discussions we discover that it is more convenient to use the bases  $B_i$ 's rather than the four actions  $\mathcal{S}_\Lambda(\Lambda^2)$ ,  $\Gamma_{1\text{st}}(\Lambda^2)$ ,  $\Gamma_{2\text{nd}}(\Lambda^2)$ , and  $\Gamma_{3\text{rd}}(\Lambda^2)$ . Therefore, we construct  $\mathcal{S}_{(3)}$  as follows,

$$\mathcal{S}_{(3)} = \mathcal{S}_{\text{WZ}} + \int d^8z \left( \sum_{i=1}^{74} B_i \right), \quad (110)$$

whose effective action is just  $\Gamma_{4\text{th}}(\Lambda^2)$ . Following the method adopted in subsections 5.6 and 5.7, we work out  $\Gamma_{4\text{th}}(\Lambda^2)$  and observe<sup>3</sup> that it no longer has new terms that do not exist in  $\mathcal{S}_{(3)}$ , i.e.,  $\Gamma_{4\text{th}}(\Lambda^2)$  can be absorbed completely by  $\mathcal{S}_{(3)}$ .

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<sup>3</sup>Because  $\Gamma_{4\text{th}}(\Lambda^2)$  takes the similar form to  $\mathcal{S}_{(3)}$ , i.e., its combinations of BFNC parameter and operator factors are covered by the 74 invariant bases  $B_i$ 's and the only difference from  $\mathcal{S}_{(3)}$  is in its coefficients, so it is not necessary for us to write it explicitly.

Now we discuss if the renormalization of parameters is compatible. The bare action is given by [16],

$$\mathcal{S}_B = \mathcal{S}_{(3)} - \Gamma_1 \Big|_{\text{dp}}, \quad (111)$$

where  $\Gamma_1|_{\text{dp}}$  equals to the effective action  $\Gamma_{4\text{th}}(\Lambda^2)$  we just mentioned.

The parameters in  $\mathcal{S}_{(3)}$  include  $m$  and  $g$  provided by action  $\mathcal{S}_{\text{WZ}}$  (eq. (14)) and  $x_{i,j}$ ,  $y_{i,j}$ , and  $z_{i,j}$  provided by the 1/2 supersymmetry invariant bases (eq. (B.2)).

We at first analyze the renormalization of fields. The following term exists in  $\Gamma_{4\text{th}}(\Lambda^2)$ ,

$$\frac{g^2}{4\pi^2\epsilon} \Phi^+ \Phi, \quad (112)$$

so the corresponding term in  $\mathcal{S}_B$  is

$$\left(1 - \frac{g^2}{4\pi^2\epsilon}\right) \Phi^+ \Phi. \quad (113)$$

The basic idea of renormalization is to redefine fields and parameters in order to transform action  $\mathcal{S}_B$  to the form of  $\mathcal{S}_{(3)}$ . We introduce  $Z$  for field renormalization,

$$Z = \left(1 - \frac{g^2}{4\pi^2\epsilon}\right), \quad (114)$$

then according to eqs. (113) and (114) redefine  $\Phi$  and  $\Phi^+$  by  $Z\Phi^+\Phi \equiv \Phi_0^+\Phi_0$ . By this way we have

$$\sqrt{Z}\Phi^+ = \Phi_0^+, \quad \sqrt{Z}\Phi = \Phi_0, \quad (115)$$

which is just same as the renormalization of fields for the ordinary Wess-Zumino model.

Now we study the renormalization of mass  $m$ . Because the following terms are not contained in  $\Gamma_{4\text{th}}(\Lambda^2)$ ,

$$\Phi \left( \frac{D^2}{\square} \Phi \right), \quad \Phi^+ \left( \frac{\bar{D}^2}{\square} \Phi^+ \right), \quad (116)$$

the renormalization of  $m$  is completely determined by the renormalization of fields. By using eq. (115), we have

$$m\Phi \left( \frac{D^2}{\square} \Phi \right) = m \left( \frac{1}{\sqrt{Z}} \right)^2 \Phi_0 \left( \frac{D^2}{\square} \Phi_0 \right). \quad (117)$$

We just need to define

$$m \left( \frac{1}{\sqrt{Z}} \right)^2 \equiv m_0, \quad (118)$$

then eq. (117) changes to be

$$m\Phi \left( \frac{D^2}{\square} \Phi \right) = m_0\Phi_0 \left( \frac{D^2}{\square} \Phi_0 \right). \quad (119)$$

Eq. (118) is the renormalization of  $m$ .

In light of eq. (115), we have for  $m\Phi^+ \left( \frac{\bar{D}^2}{\square} \Phi^+ \right)$ ,

$$m\Phi^+ \left( \frac{\bar{D}^2}{\square} \Phi^+ \right) = m \left( \frac{1}{\sqrt{Z}} \right)^2 \Phi_0^+ \left( \frac{\bar{D}^2}{\square} \Phi_0^+ \right). \quad (120)$$

In terms of the renormalization of  $m$  in eq. (118), we transform eq. (120) as follows,

$$m\Phi^+ \left( \frac{\bar{D}^2}{\square} \Phi^+ \right) = m_0 \Phi_0^+ \left( \frac{\bar{D}^2}{\square} \Phi_0^+ \right). \quad (121)$$

Therefore, we conclude that for the following two terms,

$$m\Phi \left( \frac{D^2}{\square} \Phi \right), \quad m\Phi^+ \left( \frac{\bar{D}^2}{\square} \Phi^+ \right), \quad (122)$$

the renormalization of  $m$  is same. In other words, we can say that the renormalizations of  $m$  for these two terms are compatible.

Next, we investigate the renormalization of interacting parameter  $g$ . The following terms are not contained in  $\Gamma_{4\text{th}}(\Lambda^2)$ ,

$$\Phi\Phi \left( \frac{D^2}{\square} \Phi \right), \quad \Phi^+\Phi^+ \left( \frac{\bar{D}^2}{\square} \Phi^+ \right), \quad (123)$$

so the renormalization of  $g$  is completely determined by the renormalization of fields, too. By using eq. (115), we have

$$g\Phi\Phi \left( \frac{D^2}{\square} \Phi \right) = g \left( \frac{1}{\sqrt{Z}} \right)^3 \Phi_0\Phi_0 \left( \frac{D^2}{\square} \Phi_0 \right). \quad (124)$$

We just need to define

$$g \left( \frac{1}{\sqrt{Z}} \right)^3 \equiv g_0, \quad (125)$$

then eq. (124) changes to be

$$g\Phi\Phi \left( \frac{D^2}{\square} \Phi \right) = g_0\Phi_0\Phi_0 \left( \frac{D^2}{\square} \Phi_0 \right). \quad (126)$$

Eq. (125) is the renormalization of  $g$ . The treatment is same for  $g\Phi^+\Phi^+ \left( \frac{\bar{D}^2}{\square} \Phi^+ \right)$ . In consequence, we conclude that for the following two terms,

$$g\Phi\Phi \left( \frac{D^2}{\square} \Phi \right), \quad g\Phi^+\Phi^+ \left( \frac{\bar{D}^2}{\square} \Phi^+ \right), \quad (127)$$

the renormalization of  $g$  is compatible.

At last we discuss the renormalization of parameters  $x_{i,j}$ ,  $y_{i,j}$ , and  $z_{i,j}$  that appear in the 1/2 supersymmetry invariant bases. We take  $B_{29}$  (eq. (108)) as an example. The following terms appear in the effective action  $\Gamma_{4\text{th}}(\Lambda^2)$ ,

$$B'_{29} = (-2x'_{29,3} + 2x'_{29,4}) \epsilon^{\alpha\beta} \Lambda^{kl} \theta^4 (D_\beta \Phi) \partial_l \partial_k (D_\alpha \Phi) (D^2 \Phi)$$

$$\begin{aligned}
& + \left( -6x'_{29,3} + 4x'_{29,4} + 2x'_{29,5} \right) \epsilon^{\alpha\beta} \epsilon^{\zeta\iota} \Lambda^k_{\beta} \Lambda^l_{\iota} \theta^4 \partial_k (D_{\alpha} \Phi) \partial_l (D_{\zeta} \Phi) (D^2 \Phi) \\
& + x'_{29,3} \Lambda^{kl} \theta^4 \Phi \partial_k (D^2 \Phi) \partial_l (D^2 \Phi) \\
& + x'_{29,4} \Lambda^{kl} \theta^4 \Phi (D^2 \Phi) \partial_l \partial_k (D^2 \Phi) \\
& + x'_{29,5} \epsilon^{\alpha\beta} \Lambda^{kl} \theta^4 \partial_k (D_{\alpha} \Phi) \partial_l (D_{\beta} \Phi) (D^2 \Phi), \tag{128}
\end{aligned}$$

where  $x'_{29,3}$ ,  $x'_{29,4}$ , and  $x'_{29,5}$  are linear functions of  $x_{i,j}$ ,  $y_{i,j}$ , and  $z_{i,j}$ .

For the third line of eq. (128), the corresponding terms in  $S_B$  are

$$\begin{aligned}
& x_{29,3} \Lambda^{kl} \theta^4 \Phi \partial_k (D^2 \Phi) \partial_l (D^2 \Phi) - x'_{29,3} \Lambda^{kl} \theta^4 \Phi \partial_k (D^2 \Phi) \partial_l (D^2 \Phi) \\
& = (x_{29,3} - x'_{29,3}) \Lambda^{kl} \theta^4 \Phi \partial_k (D^2 \Phi) \partial_l (D^2 \Phi) \\
& = (x_{29,3} - x'_{29,3}) \frac{\Lambda_0^{kl}}{Z_{\Lambda}} \theta^4 \frac{\Phi_0}{\sqrt{Z}} \partial_k \left( D^2 \frac{\Phi_0}{\sqrt{Z}} \right) \partial_l \left( D^2 \frac{\Phi_0}{\sqrt{Z}} \right) \\
& = x_{29,3} \left( 1 - \frac{x'_{29,3}}{x_{29,3}} \right) \frac{1}{Z_{\Lambda}} \left( \frac{1}{\sqrt{Z}} \right)^3 \Lambda_0^{kl} \theta^4 \Phi_0 \partial_k (D^2 \Phi_0) \partial_l (D^2 \Phi_0), \tag{129}
\end{aligned}$$

where we have used eq. (115) and the definition,  $\Lambda_0^{kl} \equiv Z_{\Lambda} \Lambda^{kl}$ . For analyzing renormalization we have to redefine  $x_{29,3}$  as  $(x_0)_{29,3}$  in order to transform eq. (129) into the following form,

$$(x_0)_{29,3} \Lambda_0^{kl} \theta^4 \Phi_0 \partial_k (D^2 \Phi_0) \partial_l (D^2 \Phi_0). \tag{130}$$

By using eqs. (129) and (130), we obtain

$$(x_0)_{29,3} = x_{29,3} \left( 1 - \frac{x'_{29,3}}{x_{29,3}} \right) \frac{1}{Z_{\Lambda}} \left( \frac{1}{\sqrt{Z}} \right)^3. \tag{131}$$

This is the renormalization of parameter  $x_{29,3}$ .

Because both  $B_{29}$  and  $B'_{29}$  are 1/2 supersymmetry invariant bases and the numbers of their independent parameters are same, the renormalization of their parameters is compatible. The similar discussion can be applied to all of the parameters  $x_{i,j}$ ,  $y_{i,j}$ , and  $z_{i,j}$  of the 1/2 supersymmetry invariant bases. As a result, we prove that the renormalization of all the parameter  $m$ ,  $g$ ,  $x_{i,j}$ ,  $y_{i,j}$ , and  $z_{i,j}$  is compatible.<sup>4</sup>

Finally, we can conclude that the action  $\mathcal{S}_{(3)}$  is one-loop renormalizable.

## 6 Conclusion and Outlook

Through analyzing the Wess-Zumino model on the BFNC superspace, we know that the action obtained by replacing the ordinary product by the star product is not renormalizable in general. To

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<sup>4</sup>We note that in ref. [16] the same parameters are used in different terms of actions. Such a treatment is doubtful since there no symmetries ensure that the parameters are compatible for renormalization.



make it renormalizable one should add to it new terms. For the NAC superspace, which is a simpler case, one just needs to add the terms from the primary one-loop effective action, and then provides the renormalizable action to all orders in perturbation theory [12]. However, for the BFNC superspace the situation is much more complicated. The iterative process should go up to the third time. Moreover, the complexity also includes that the obtained renormalizable action has so many terms that can be classified on the one hand into 74 subsets each of which has the 1/2 supersymmetry invariance, and on the other hand into 74 bases that correspond to the 74 subsets. In particular, in light of the invariant bases we construct the one-loop renormalizable action (eq. (110)) up to the second order of the BFNC parameters  $\Lambda^{k\alpha}$ 's.

One of our further considerations is to investigate the all order loop renormalization of the action eq. (110). In addition, we shall extend the above analysis to all orders of the BFNC parameters  $\Lambda^{k\alpha}$ 's. On the other hand, we shall explore the super Yang-Mills model on the BFNC superspace and search for its renormalizable formulation by following the way we have utilized for the Wess-Zumino model.

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# Appendix

## A Algebraic Relations

The following identities of  $\sigma$  algebra are used in our calculations of effective actions.

$$\begin{aligned}
\epsilon^{klmn}\eta_{kl} &= 0, & \theta^\alpha\theta^\beta\theta^\gamma &= 0, \\
\left(\bar{\sigma}^{kl}\right)_{\dot{\alpha}}^{\dot{\alpha}} &= 0, & \left(\sigma^{kl}\right)_\alpha^\alpha &= 0, & \left(\bar{\sigma}^{kk}\right)_{\dot{\beta}}^{\dot{\alpha}} &= 0, & \left(\sigma^{kk}\right)_\alpha^\beta &= 0, \\
\delta_{\alpha\alpha} &= 2, & \delta_{kk} &= 4, & \eta^{kl}\eta_{lm} &= \delta_{km}, & \epsilon^{\alpha\beta}\epsilon_{\beta\gamma} &= \delta_{\alpha\gamma}, \\
\theta_\alpha\theta_\beta &= \frac{1}{2}\epsilon_{\alpha\beta}\theta^\zeta\theta_\zeta, & \bar{\theta}_{\dot{\gamma}}\bar{\theta}_{\dot{\kappa}} &= -\frac{1}{2}\epsilon_{\dot{\gamma}\dot{\kappa}}\bar{\theta}_{\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}, \\
\sigma^l_{\beta\dot{\gamma}}\left(\bar{\sigma}^k\right)^{\dot{\alpha}\beta} &= -\delta_{\dot{\alpha}\dot{\gamma}}\eta^{kl} + 2\left(\bar{\sigma}^{kl}\right)^{\dot{\alpha}}_{\dot{\gamma}}, \\
\sigma^k_{\alpha\dot{\beta}}\left(\bar{\sigma}^l\right)^{\dot{\beta}\gamma} &= -\delta_{\alpha\gamma}\eta^{kl} + 2\left(\sigma^{kl}\right)_\alpha^\gamma, \\
\sigma^k_{\gamma\dot{\zeta}}\left(\bar{\sigma}^{lm}\right)^{\dot{\zeta}}_{\dot{\alpha}} &= \frac{1}{2}\eta^{km}\sigma^l_{\gamma\dot{\alpha}} - \frac{1}{2}\eta^{kl}\sigma^m_{\gamma\dot{\alpha}} - \frac{1}{2}i\eta_{no}\epsilon^{oklm}\sigma^n_{\gamma\dot{\alpha}}, \\
\left(\bar{\sigma}^m\right)^{\dot{\beta}\gamma}\left(\bar{\sigma}^{kl}\right)^{\dot{\alpha}}_{\dot{\beta}} &= -\frac{1}{2}\eta^{lm}\left(\bar{\sigma}^k\right)^{\dot{\alpha}\gamma} + \frac{1}{2}\eta^{km}\left(\bar{\sigma}^l\right)^{\dot{\alpha}\gamma} + \frac{1}{2}i\eta_{no}\epsilon^{oklm}\left(\bar{\sigma}^n\right)^{\dot{\alpha}\gamma}, \\
\left(\bar{\sigma}^m\right)^{\dot{\alpha}\beta}\left(\sigma^{kl}\right)_\beta^\gamma &= \frac{1}{2}\eta^{lm}\left(\bar{\sigma}^k\right)^{\dot{\alpha}\gamma} - \frac{1}{2}\eta^{km}\left(\bar{\sigma}^l\right)^{\dot{\alpha}\gamma} + \frac{1}{2}i\eta_{no}\epsilon^{oklm}\left(\bar{\sigma}^n\right)^{\dot{\alpha}\gamma}, \\
\sigma^m_{\beta\dot{\gamma}}\left(\sigma^{kl}\right)_\alpha^\beta &= -\frac{1}{2}\eta^{lm}\sigma^k_{\alpha\dot{\gamma}} + \frac{1}{2}\eta^{km}\sigma^l_{\alpha\dot{\gamma}} - \frac{1}{2}i\eta_{no}\epsilon^{oklm}\sigma^n_{\alpha\dot{\gamma}}, \\
\left(\bar{\sigma}^{kl}\right)^{\dot{\alpha}}_{\dot{\beta}}\left(\bar{\sigma}^{mn}\right)^{\dot{\beta}}_{\dot{\alpha}} &= \frac{1}{2}i\epsilon^{klmn} + \frac{1}{2}\eta^{kn}\eta^{lm} - \frac{1}{2}\eta^{km}\eta^{ln}, \\
\left(\sigma^{kl}\right)_\alpha^\zeta\left(\sigma^{mn}\right)_\zeta^\alpha &= -\frac{1}{2}i\epsilon^{klmn} + \frac{1}{2}\eta^{kn}\eta^{lm} - \frac{1}{2}\eta^{km}\eta^{ln}, \\
\Lambda^{kl}\Lambda^{mn}\partial_k\partial_l\partial_m X &= 0, & \Lambda^{kl}\Lambda^{m\alpha}\partial_k\partial_l\partial_m X &= 0, \\
\left(\sigma^{kl}\right)_\alpha^\beta\partial_k\partial_l X &= 0, & \left(\sigma^{kl}\right)_\alpha^\beta\Lambda^{n\alpha}\Lambda^o_\beta\partial_n\partial_o X &= 0, \\
\Lambda^{k\alpha}\Lambda^{l\beta}\partial_k\partial_l X &= -\frac{1}{2}\epsilon^{\alpha\beta}\Lambda^{kl}\partial_k\partial_l X, & \left(\sigma^{kl}\right)_\alpha^\beta\Lambda^{n\alpha}\Lambda^m_\beta &= -\left(\sigma^{kl}\right)_\alpha^\beta\Lambda^{m\alpha}\Lambda^n_\beta, \\
\Lambda^{kl}\Lambda^{mn}\partial_k\partial_m X &= -\frac{1}{2}\Lambda^{km}\Lambda^{ln}\partial_k\partial_m X, & \Lambda^{kl}\Lambda^{m\alpha}\partial_k\partial_m X &= -\frac{1}{2}\Lambda^{km}\Lambda^{l\alpha}\partial_k\partial_m X,
\end{aligned} \tag{A1}$$

where  $X$  stands for a superfield.

Moreover, the following relations of  $D$  algebra are also used.

$$\begin{aligned}
\left(\bar{D}_{\dot{\beta}}D_\alpha\Phi\right) &= -2i\sigma^k_{\alpha\dot{\beta}}(\partial_k\Phi), & \left(\bar{D}^2D_\alpha\Phi\right) &= 0, \\
\left(\bar{D}_{\dot{\alpha}}D^2\Phi\right) &= -4i\epsilon^{\beta\zeta}\sigma^k_{\zeta\dot{\alpha}}(\partial_kD_\beta\Phi), & \left(\bar{D}^2D^2\Phi\right) &= 16(\square\Phi), \\
\left(D_\alpha\bar{D}_{\dot{\beta}}\Phi^+\right) &= -2i\sigma^k_{\alpha\dot{\beta}}(\partial_k\Phi^+), & \left(D^2\bar{D}_{\dot{\alpha}}\Phi^+\right) &= 0, \\
\left(D_\alpha\bar{D}^2\Phi^+\right) &= 4i\epsilon^{\dot{\beta}\dot{\zeta}}\sigma^k_{\alpha\dot{\zeta}}(\partial_k\bar{D}_{\dot{\beta}}\Phi^+), & \left(D^2\bar{D}^2\Phi^+\right) &= 16(\square\Phi^+), \\
D^2\bar{D}^2D^2 &= 16\square D^2, & D^2\bar{D}^2D_\alpha &= -4i\epsilon^{\dot{\beta}\dot{\zeta}}p\sigma_{\alpha\dot{\zeta}}D^2\bar{D}_{\dot{\beta}}, \\
\bar{D}^2D^2\bar{D}_{\dot{\alpha}} &= 4i\epsilon^{\beta\zeta}p\sigma_{\zeta\dot{\alpha}}\bar{D}^2D_\beta, & \bar{D}^2D_\alpha\bar{D}_{\dot{\beta}} &= -2ip\sigma_{\alpha\dot{\beta}}\bar{D}^2,
\end{aligned}$$

$$\begin{aligned}
D_\alpha \bar{D}^2 D^2 &= 4i\epsilon^{\dot{\beta}\dot{\zeta}} p\sigma_{\alpha\dot{\zeta}} \bar{D}_{\dot{\beta}} D^2, & D_\alpha \bar{D}_{\dot{\beta}} D^2 &= -2ip\sigma_{\alpha\dot{\beta}} D^2, \\
\bar{D}_{\dot{\alpha}} D^2 \bar{D}^2 &= -4i\epsilon^{\beta\zeta} p\sigma_{\zeta\dot{\alpha}} D_\beta \bar{D}^2, & \bar{D}_{\dot{\alpha}} D_\beta \bar{D}^2 &= -2ip\sigma_{\beta\dot{\alpha}} \bar{D}^2, \\
D^2 \bar{D}_\alpha D_\beta &= -2ip\sigma_{\beta\dot{\alpha}} D^2, & \bar{D}^2 D^2 \bar{D}^2 &= 16\Box \bar{D}^2, \\
D_\alpha \bar{D}^2 D_\beta &= \frac{1}{2}\epsilon_{\alpha\beta} D^2 \bar{D}^2 - 4i\epsilon^{\dot{\zeta}i} p\sigma_{\beta i} D_\alpha \bar{D}_{\dot{\zeta}}, \\
\bar{D}_{\dot{\alpha}} D^2 \bar{D}_{\dot{\beta}} &= -\frac{1}{2}\epsilon_{\dot{\alpha}\dot{\beta}} D^2 \bar{D}^2 - 4i\epsilon^{\zeta i} p\sigma_{i\dot{\alpha}} D_\zeta \bar{D}_{\dot{\beta}}, \\
D_\alpha \bar{D}_{\dot{\beta}} D_\zeta &= -2ip\sigma_{\zeta\dot{\beta}} D_\alpha - \frac{1}{2}\epsilon_{\alpha\zeta} D^2 \bar{D}_{\dot{\beta}}, \\
\bar{D}_{\dot{\alpha}} D_\beta \bar{D}_{\dot{\zeta}} &= \frac{1}{2}\epsilon_{\dot{\alpha}\dot{\zeta}} \bar{D}^2 D_\beta - 2ip\sigma_{\beta\dot{\zeta}} \bar{D}_{\dot{\alpha}},
\end{aligned} \tag{A2}$$

where the identities for  $p\Lambda$  and  $p\sigma$  read

$$\begin{aligned}
p\Lambda_\alpha p\Lambda_\beta &= \frac{1}{2}\epsilon_{\alpha\beta} p\Lambda^2, & \epsilon^{\alpha\beta} \epsilon^{\dot{\gamma}\dot{\delta}} p\sigma_{\beta\dot{\delta}} &= \overline{p\sigma}^{\dot{\gamma}\alpha}, \\
\epsilon_{\alpha\beta} \epsilon_{\dot{\gamma}\dot{\delta}} \overline{p\sigma}^{\dot{\delta}\beta} &= p\sigma_{\alpha\dot{\gamma}}, & \epsilon_{\dot{\alpha}\dot{\beta}} \overline{p\sigma}^{\dot{\beta}i} \overline{p\sigma}^{\dot{\alpha}\zeta} &= \epsilon^{\zeta i} \Box, \\
\epsilon_{\zeta i} \overline{p\sigma}^{\dot{\beta}i} \overline{p\sigma}^{\dot{\alpha}\zeta} &= \epsilon^{\dot{\alpha}\dot{\beta}} \Box, & \epsilon^{\alpha\zeta} p\sigma_{\alpha\dot{\beta}} p\sigma_{\zeta i} &= \epsilon_{\dot{\beta}i} \Box, \\
\epsilon^{\dot{\beta}i} p\sigma_{\alpha\dot{\beta}} p\sigma_{\zeta i} &= \epsilon_{\alpha\zeta} \Box, & p\sigma_{\beta\dot{\gamma}} \overline{p\sigma}^{\dot{\alpha}\beta} &= -\delta_{\dot{\alpha}\dot{\gamma}} \Box, \\
\overline{p\sigma}^{\dot{\alpha}\beta} p\sigma_{\beta\dot{\gamma}} &= -\delta_{\dot{\alpha}\dot{\gamma}} \Box.
\end{aligned} \tag{A3}$$

## B Effective Actions

### B.1 Supersymmetry Invariant Subsets

We list the 74 subsets  $f_i$ 's below, each of them is invariant under the  $1/2$  supersymmetry transformation.

$$\begin{aligned}
f_1 &= \frac{1}{512} g^4 m^4 (3\Lambda^2 + 2\sigma\Lambda\Lambda) (10\mathbf{a}_2 - 71g^2\mathbf{a}_3) \epsilon^{\alpha\beta} \theta^4 (D_\alpha \Phi) (D_\beta \Phi) \\
&\quad + \frac{1}{512} g^4 m^4 (3\Lambda^2 + 2\sigma\Lambda\Lambda) (-10\mathbf{a}_2 + 71g^2\mathbf{a}_3) \theta^4 \Phi (D^2 \Phi), \\
f_2 &= \frac{1}{256} g^5 m^3 (3\Lambda^2 + 2\sigma\Lambda\Lambda) (10\mathbf{a}_2 - 71g^2\mathbf{a}_3) \epsilon^{\alpha\beta} \theta^4 \Phi (D_\alpha \Phi) (D_\beta \Phi) \\
&\quad + \frac{1}{512} g^5 m^3 (3\Lambda^2 + 2\sigma\Lambda\Lambda) (-10\mathbf{a}_2 + 71g^2\mathbf{a}_3) \theta^4 \Phi \Phi (D^2 \Phi), \\
f_3 &= \frac{3}{256} g^5 m^3 (3\Lambda^2 + 2\sigma\Lambda\Lambda) (10\mathbf{a}_2 - 71g^2\mathbf{a}_3) \epsilon^{\alpha\beta} \theta^4 (D_\alpha \Phi) (D_\beta \Phi) \Phi^+ \\
&\quad + \frac{3}{256} g^5 m^3 (3\Lambda^2 + 2\sigma\Lambda\Lambda) (-10\mathbf{a}_2 + 71g^2\mathbf{a}_3) \theta^4 \Phi (D^2 \Phi) \Phi^+, \\
f_4 &= \frac{3}{128} g^6 m^2 (3\Lambda^2 + 2\sigma\Lambda\Lambda) (10\mathbf{a}_2 - 71g^2\mathbf{a}_3) \epsilon^{\alpha\beta} \theta^4 \Phi (D_\alpha \Phi) (D_\beta \Phi) \Phi^+ \\
&\quad + \frac{3}{256} g^6 m^2 (3\Lambda^2 + 2\sigma\Lambda\Lambda) (-10\mathbf{a}_2 + 71g^2\mathbf{a}_3) \theta^4 \Phi \Phi (D^2 \Phi) \Phi^+, \\
f_5 &= \frac{3}{128} g^6 m^2 (3\Lambda^2 + 2\sigma\Lambda\Lambda) (10\mathbf{a}_2 - 71g^2\mathbf{a}_3) \epsilon^{\alpha\beta} \theta^4 (D_\alpha \Phi) (D_\beta \Phi) \Phi^+ \Phi^+ \\
&\quad + \frac{3}{128} g^6 m^2 (3\Lambda^2 + 2\sigma\Lambda\Lambda) (-10\mathbf{a}_2 + 71g^2\mathbf{a}_3) \theta^4 \Phi (D^2 \Phi) \Phi^+ \Phi^+, \\
f_6 &= \frac{3}{64} g^7 m (3\Lambda^2 + 2\sigma\Lambda\Lambda) (10\mathbf{a}_2 - 71g^2\mathbf{a}_3) \epsilon^{\alpha\beta} \theta^4 \Phi (D_\alpha \Phi) (D_\beta \Phi) \Phi^+ \Phi^+
\end{aligned}$$

$$\begin{aligned}
& + \frac{3}{128} g^7 m (3\Lambda^2 + 2\sigma\Lambda\Lambda) (-10\mathbf{a}_2 + 71g^2\mathbf{a}_3) \theta^4 \Phi \Phi (D^2\Phi) \Phi^+ \Phi^+, \\
f_7 &= \frac{1}{64} g^7 m (3\Lambda^2 + 2\sigma\Lambda\Lambda) (10\mathbf{a}_2 - 71g^2\mathbf{a}_3) \epsilon^{\alpha\beta} \theta^4 (D_\alpha \Phi) (D_\beta \Phi) \Phi^+ \Phi^+ \Phi^+ \\
& + \frac{1}{64} g^7 m (3\Lambda^2 + 2\sigma\Lambda\Lambda) (-10\mathbf{a}_2 + 71g^2\mathbf{a}_3) \theta^4 \Phi (D^2\Phi) \Phi^+ \Phi^+ \Phi^+, \\
f_8 &= \frac{1}{32} g^8 (3\Lambda^2 + 2\sigma\Lambda\Lambda) (10\mathbf{a}_2 - 71g^2\mathbf{a}_3) \epsilon^{\alpha\beta} \theta^4 \Phi (D_\alpha \Phi) (D_\beta \Phi) \Phi^+ \Phi^+ \Phi^+ \\
& + \frac{1}{64} g^8 (3\Lambda^2 + 2\sigma\Lambda\Lambda) (-10\mathbf{a}_2 + 71g^2\mathbf{a}_3) \theta^4 \Phi \Phi (D^2\Phi) \Phi^+ \Phi^+ \Phi^+, \\
f_9 &= \frac{1}{512} g^5 m^2 (3\Lambda^2 + 2\sigma\Lambda\Lambda) (2\mathbf{a}_2 - 15g^2\mathbf{a}_3) \epsilon^{\alpha\beta} \theta^4 (D_\alpha \Phi) (D_\beta \Phi) (D^2\Phi) \\
& + \frac{1}{512} g^5 m^2 (3\Lambda^2 + 2\sigma\Lambda\Lambda) (-2\mathbf{a}_2 + 15g^2\mathbf{a}_3) \theta^4 \Phi (D^2\Phi) (D^2\Phi), \\
f_{10} &= \frac{1}{256} g^6 m (3\Lambda^2 + 2\sigma\Lambda\Lambda) (2\mathbf{a}_2 - 15g^2\mathbf{a}_3) \epsilon^{\alpha\beta} \theta^4 \Phi (D_\alpha \Phi) (D_\beta \Phi) (D^2\Phi) \\
& + \frac{1}{512} g^6 m (3\Lambda^2 + 2\sigma\Lambda\Lambda) (-2\mathbf{a}_2 + 15g^2\mathbf{a}_3) \theta^4 \Phi \Phi (D^2\Phi) (D^2\Phi), \\
f_{11} &= \frac{1}{256} g^6 m (3\Lambda^2 + 2\sigma\Lambda\Lambda) (2\mathbf{a}_2 - 15g^2\mathbf{a}_3) \epsilon^{\alpha\beta} \theta^4 (D_\alpha \Phi) (D_\beta \Phi) (D^2\Phi) \Phi^+ \\
& + \frac{1}{256} g^6 m (3\Lambda^2 + 2\sigma\Lambda\Lambda) (-2\mathbf{a}_2 + 15g^2\mathbf{a}_3) \theta^4 \Phi (D^2\Phi) (D^2\Phi) \Phi^+, \\
f_{12} &= \frac{1}{128} g^7 (3\Lambda^2 + 2\sigma\Lambda\Lambda) (2\mathbf{a}_2 - 15g^2\mathbf{a}_3) \epsilon^{\alpha\beta} \theta^4 \Phi (D_\alpha \Phi) (D_\beta \Phi) (D^2\Phi) \Phi^+ \\
& + \frac{1}{256} g^7 (3\Lambda^2 + 2\sigma\Lambda\Lambda) (-2\mathbf{a}_2 + 15g^2\mathbf{a}_3) \theta^4 \Phi \Phi (D^2\Phi) (D^2\Phi) \Phi^+, \\
f_{13} &= \frac{1}{64} i g^5 m^2 (3\Lambda^2 - 2\sigma\Lambda\Lambda) (\mathbf{a}_2 - 4g^2\mathbf{a}_3) (\bar{\sigma}^k)^{\dot{\alpha}\beta} \theta^4 (D_\beta \Phi) \partial_k \Phi^+ (\bar{D}_{\dot{\alpha}} \Phi^+) \\
& + \frac{1}{32} g^5 m^2 (3\Lambda^2 - 2\sigma\Lambda\Lambda) (-\mathbf{a}_2 + 4g^2\mathbf{a}_3) \eta^{kl} \theta^4 \Phi \partial_k \Phi^+ \partial_l \Phi^+ \\
& + \frac{1}{512} g^5 m^2 (3\Lambda^2 - 2\sigma\Lambda\Lambda) (-\mathbf{a}_2 + 4g^2\mathbf{a}_3) \epsilon^{\dot{\alpha}\dot{\beta}} \theta^4 (D^2\Phi) (\bar{D}_{\dot{\alpha}} \Phi^+) (\bar{D}_{\dot{\beta}} \Phi^+), \\
f_{14} &= \frac{1}{16} i g^6 m (3\Lambda^2 - 2\sigma\Lambda\Lambda) (\mathbf{a}_2 - 4g^2\mathbf{a}_3) (\bar{\sigma}^k)^{\dot{\alpha}\beta} \theta^4 (D_\beta \Phi) \Phi^+ \partial_k \Phi^+ (\bar{D}_{\dot{\alpha}} \Phi^+) \\
& + \frac{1}{8} g^6 m (3\Lambda^2 - 2\sigma\Lambda\Lambda) (-\mathbf{a}_2 + 4g^2\mathbf{a}_3) \eta^{kl} \theta^4 \Phi \Phi^+ \partial_k \Phi^+ \partial_l \Phi^+ \\
& + \frac{1}{128} g^6 m (3\Lambda^2 - 2\sigma\Lambda\Lambda) (-\mathbf{a}_2 + 4g^2\mathbf{a}_3) \epsilon^{\dot{\alpha}\dot{\beta}} \theta^4 (D^2\Phi) \Phi^+ (\bar{D}_{\dot{\alpha}} \Phi^+) (\bar{D}_{\dot{\beta}} \Phi^+), \\
f_{15} &= \frac{1}{16} i g^7 (3\Lambda^2 - 2\sigma\Lambda\Lambda) (\mathbf{a}_2 - 4g^2\mathbf{a}_3) (\bar{\sigma}^k)^{\dot{\alpha}\beta} \theta^4 (D_\beta \Phi) \Phi^+ \Phi^+ \partial_k \Phi^+ (\bar{D}_{\dot{\alpha}} \Phi^+) \\
& + \frac{1}{8} g^7 (3\Lambda^2 - 2\sigma\Lambda\Lambda) (-\mathbf{a}_2 + 4g^2\mathbf{a}_3) \eta^{kl} \theta^4 \Phi \Phi^+ \Phi^+ \partial_k \Phi^+ \partial_l \Phi^+ \\
& + \frac{1}{128} g^7 (3\Lambda^2 - 2\sigma\Lambda\Lambda) (-\mathbf{a}_2 + 4g^2\mathbf{a}_3) \epsilon^{\dot{\alpha}\dot{\beta}} \theta^4 (D^2\Phi) \Phi^+ \Phi^+ (\bar{D}_{\dot{\alpha}} \Phi^+) (\bar{D}_{\dot{\beta}} \Phi^+), \\
f_{16} &= \frac{g^4 m^2 (-4\mathbf{a}_2 + 23g^2\mathbf{a}_3)}{2304} (\eta\sigma\Lambda\Lambda^k)^l \theta^4 \Phi \partial_k \partial_l (D^2\Phi) \\
& + \frac{g^4 m^2 (-4\mathbf{a}_2 + 23g^2\mathbf{a}_3)}{2304} \epsilon^{\alpha\beta} (\eta\sigma\Lambda\Lambda^k)^l \theta^4 (D_\beta \Phi) \partial_k \partial_l (D_\alpha \Phi), \\
f_{17} &= \frac{1}{288} g^5 m \mathbf{a}_2 \epsilon^{\alpha\beta} (\eta\sigma\Lambda^k)^\zeta \Lambda^l{}_\beta \theta^4 (D_\alpha \Phi) \partial_k \partial_l (D_\zeta \Phi) \Phi^+ \\
& - \frac{1}{36} g^5 m \mathbf{a}_2 \epsilon^{\alpha\beta} (\eta\sigma\Lambda^k)^\zeta \Lambda^l{}_\beta \theta^4 \partial_k (D_\alpha \Phi) \partial_l (D_\zeta \Phi) \Phi^+ \\
& + \frac{ig^5 m (-4\mathbf{a}_2 + 23g^2\mathbf{a}_3)}{9216} \eta_{kl} (\bar{\sigma}^l)^{\dot{\alpha}\beta} (\eta\sigma\Lambda\Lambda^n)^k \theta^4 (D_\beta \Phi) \partial_n (D^2\Phi) (\bar{D}_{\dot{\alpha}} \Phi^+) \\
& - \frac{ig^5 m (-4\mathbf{a}_2 + 23g^2\mathbf{a}_3)}{9216} \eta_{kl} (\bar{\sigma}^l)^{\dot{\alpha}\beta} (\eta\sigma\Lambda\Lambda^n)^k \theta^4 \partial_n (D_\beta \Phi) (D^2\Phi) (\bar{D}_{\dot{\alpha}} \Phi^+) \\
& - \frac{ig^5 m (-4\mathbf{a}_2 + 23g^2\mathbf{a}_3)}{4608} \epsilon^{\alpha\beta} \epsilon^{\zeta\iota} \epsilon^{klno} \eta_{np} \eta_{oq} \Lambda^p{}_\beta \Lambda^q{}_\iota \theta^4 \partial_k (D_\alpha \Phi) \partial_l (D_\zeta \Phi) \Phi^+ \\
& + \frac{g^5 m (28\mathbf{a}_2 + 23g^2\mathbf{a}_3)}{1152} \Lambda^{kl} \eta_{ln} (\sigma^{no})^{\alpha\beta} \theta^4 \partial_k (D_\alpha \Phi) \partial_o (D_\beta \Phi) \Phi^+ \\
& + \frac{g^5 m (-124\mathbf{a}_2 + 129g^2\mathbf{a}_3)}{4608} \epsilon^{\alpha\beta} \Lambda^{kl} \theta^4 \partial_k (D_\alpha \Phi) \partial_l (D_\beta \Phi) \Phi^+ \\
& + \frac{1}{288} g^3 m (9\mathbf{a}_1 - 6g^2\mathbf{a}_2 + 17g^4\mathbf{a}_3) \Lambda^{kl} \theta^4 \partial_l \Phi \partial_k (D^2\Phi) \Phi^+ \\
& + \frac{1}{288} g^3 m (27\mathbf{a}_1 - 37g^2\mathbf{a}_2 + 69g^4\mathbf{a}_3) \Lambda^{kl} \theta^4 \partial_k \partial_l \Phi (D^2\Phi) \Phi^+
\end{aligned}$$

$$\begin{aligned}
& -\frac{ig^3m(36\mathbf{a}_1-52g^2\mathbf{a}_2+83g^4\mathbf{a}_3)}{4608}\Lambda^{kl}\eta_{ln}(\bar{\sigma}^n)^{\dot{\alpha}\beta}\theta^4(D_\beta\Phi)\partial_k(D^2\Phi)(\bar{D}_{\dot{\alpha}}\Phi^+) \\
& +\frac{ig^3m(36\mathbf{a}_1-52g^2\mathbf{a}_2+83g^4\mathbf{a}_3)}{4608}\Lambda^{kl}\eta_{ln}(\bar{\sigma}^n)^{\dot{\alpha}\beta}\theta^4\partial_k(D_\beta\Phi)(D^2\Phi)(\bar{D}_{\dot{\alpha}}\Phi^+) \\
& +\frac{1}{576}g^3m(36\mathbf{a}_1-40g^2\mathbf{a}_2+87g^4\mathbf{a}_3)\left(\eta\sigma\Lambda\Lambda^k\right)^l\theta^4\partial_k\partial_l(D^2\Phi)\Phi^+ \\
& -\frac{1}{288}g^3m(36\mathbf{a}_1-57g^2\mathbf{a}_2+106g^4\mathbf{a}_3)\epsilon^{\alpha\beta}\eta_{kl}\left(\sigma\Lambda^{ln}\right)^{o\zeta}\Lambda^k{}_\beta\theta^4(D_\alpha\Phi)\partial_n\partial_o(D_\zeta\Phi)\Phi^+ \\
& -\frac{1}{576}g^3m(72\mathbf{a}_1-92g^2\mathbf{a}_2+189g^4\mathbf{a}_3)\epsilon^{\alpha\beta}\eta_{kl}\left(\sigma\Lambda^{ln}\right)^{o\zeta}\Lambda^k{}_\beta\theta^4\partial_n(D_\alpha\Phi)\partial_o(D_\zeta\Phi)\Phi^+ \\
& +\frac{1}{576}g^3m(72\mathbf{a}_1-96g^2\mathbf{a}_2+193g^4\mathbf{a}_3)\left(\eta\sigma\Lambda\Lambda^k\right)^l\theta^4\Phi\partial_k\partial_l(D^2\Phi)\Phi^+ \\
& +\frac{g^3m(72\mathbf{a}_1-96g^2\mathbf{a}_2+193g^4\mathbf{a}_3)}{1152}\Lambda^{kl}\theta^4\Phi\partial_l\partial_k(D^2\Phi)\Phi^+ \\
& +\frac{1}{288}g^3m(72\mathbf{a}_1-96g^2\mathbf{a}_2+193g^4\mathbf{a}_3)\epsilon^{\alpha\beta}\left(\eta\sigma\Lambda\Lambda^k\right)^l\theta^4(D_\beta\Phi)\partial_k\partial_l(D_\alpha\Phi)\Phi^+, \\
& +\frac{g^3m(144\mathbf{a}_1-156g^2\mathbf{a}_2+325g^4\mathbf{a}_3)}{2304}\left(\eta\sigma\Lambda\Lambda^k\right)^l\theta^4\partial_l\Phi\partial_k(D^2\Phi)\Phi^+ \\
& +\frac{ig^3m(144\mathbf{a}_1-220g^2\mathbf{a}_2+401g^4\mathbf{a}_3)}{9216}\eta_{kl}\left(\bar{\sigma}^l\right)^{\dot{\alpha}\beta}\left(\eta\sigma\Lambda\Lambda^k\right)^n\theta^4(D_\beta\Phi)\partial_n(D^2\Phi)(\bar{D}_{\dot{\alpha}}\Phi^+) \\
& -\frac{ig^3m(144\mathbf{a}_1-220g^2\mathbf{a}_2+401g^4\mathbf{a}_3)}{9216}\eta_{kl}\left(\bar{\sigma}^l\right)^{\dot{\alpha}\beta}\left(\eta\sigma\Lambda\Lambda^k\right)^n\theta^4\partial_n(D_\beta\Phi)(D^2\Phi)(\bar{D}_{\dot{\alpha}}\Phi^+) \\
& +\frac{g^3m(144\mathbf{a}_1-220g^2\mathbf{a}_2+401g^4\mathbf{a}_3)}{2304}\epsilon^{\alpha\beta}\epsilon^{\zeta\iota}\Lambda^k{}_\beta\Lambda^l{}_\iota\theta^4\partial_k(D_\alpha\Phi)\partial_l(D_\zeta\Phi)\Phi^+ \\
& +\frac{g^3m(180\mathbf{a}_1-244g^2\mathbf{a}_2+469g^4\mathbf{a}_3)}{1152}\epsilon^{\alpha\beta}\Lambda^{kl}\theta^4(D_\beta\Phi)\partial_l\partial_k(D_\alpha\Phi)\Phi^+ \\
& +\frac{g^3m(288\mathbf{a}_1-372g^2\mathbf{a}_2+703g^4\mathbf{a}_3)}{2304}\left(\eta\sigma\Lambda\Lambda^k\right)^l\theta^4\partial_k\Phi\partial_l(D^2\Phi)\Phi^+ \\
& +\frac{g^3m(288\mathbf{a}_1-372g^2\mathbf{a}_2+703g^4\mathbf{a}_3)}{1152}\epsilon^{\alpha\beta}\left(\eta\sigma\Lambda\Lambda^k\right)^l\theta^4\partial_k(D_\beta\Phi)\partial_l(D_\alpha\Phi)\Phi^+ \\
& +\frac{1}{2304}(4g^5m\mathbf{a}_2-23g^7m\mathbf{a}_3)\epsilon^{\alpha\beta}\eta_{kl}\left(\eta\sigma\Lambda^l\right)^\zeta\Lambda^k{}_\beta\theta^4(D_\alpha\Phi)\square(D_\zeta\Phi)\Phi^+ \\
& +\frac{1}{576}(6g^5m\mathbf{a}_2-23g^7m\mathbf{a}_3)\Lambda^{kl}\eta_{ln}(\sigma^{no})^{\alpha\beta}\theta^4(D_\alpha\Phi)\partial_k\partial_o(D_\beta\Phi)\Phi^+ \\
& +\frac{g^3m(288\Lambda^2\mathbf{a}_1-16g^2(22\Lambda^2-7\sigma\Lambda\Lambda)\mathbf{a}_2+3g^4(407\Lambda^2-122\sigma\Lambda\Lambda)\mathbf{a}_3)}{9216}\theta^4\square\Phi(D^2\Phi)\Phi^+ \\
& +\frac{g^3m(48\Lambda^2\mathbf{a}_1-8g^2(15\Lambda^2+2\sigma\Lambda\Lambda)\mathbf{a}_2+3g^4(127\Lambda^2+18\sigma\Lambda\Lambda)\mathbf{a}_3)}{49152}\theta^4(D^2\Phi)(D^2\Phi)(\bar{D}^2\Phi^+) \\
& +\frac{g^5m(148\Lambda^2\mathbf{a}_2+g^2(-689\Lambda^2+252\sigma\Lambda\Lambda)\mathbf{a}_3)}{4608}\left(\sigma^{kl}\right)^{\alpha\beta}\theta^4\partial_k(D_\alpha\Phi)\partial_l(D_\beta\Phi)\Phi^+ \\
& +\frac{g^3m(72\Lambda^2\mathbf{a}_1+g^2((-51\Lambda^2+30\sigma\Lambda\Lambda)\mathbf{a}_2+g^2(133\Lambda^2-40\sigma\Lambda\Lambda)\mathbf{a}_3))}{2304}\theta^4\Phi\square(D^2\Phi)\Phi^+ \\
& +\frac{g^3m(576\Lambda^2\mathbf{a}_1+g^2((-636\Lambda^2+72\sigma\Lambda\Lambda)\mathbf{a}_2+g^2(1565\Lambda^2+106\sigma\Lambda\Lambda)\mathbf{a}_3))}{9216}\eta^{kl}\theta^4\partial_l\Phi\partial_k(D^2\Phi)\Phi^+ \\
& +\frac{ig^3m(288\Lambda^2\mathbf{a}_1+g^2(-4(217\Lambda^2+26\sigma\Lambda\Lambda)\mathbf{a}_2+g^2(2975\Lambda^2+118\sigma\Lambda\Lambda)\mathbf{a}_3))}{36864}\left(\bar{\sigma}^k\right)^{\dot{\alpha}\beta}\theta^4\partial_k(D_\beta\Phi)(D^2\Phi)(\bar{D}_{\dot{\alpha}}\Phi^+) \\
& +\frac{g^3m(-288\Lambda^2\mathbf{a}_1+g^2(-84(\Lambda^2+2\sigma\Lambda\Lambda)\mathbf{a}_2+g^2(721\Lambda^2+218\sigma\Lambda\Lambda)\mathbf{a}_3))}{9216}\eta^{kl}\epsilon^{\alpha\beta}\theta^4\partial_k(D_\alpha\Phi)\partial_l(D_\beta\Phi)\Phi^+ \\
& +\frac{ig^3m(288\Lambda^2\mathbf{a}_1+g^2(-44(13\Lambda^2+2\sigma\Lambda\Lambda)\mathbf{a}_2+g^2(1597\Lambda^2+530\sigma\Lambda\Lambda)\mathbf{a}_3))}{36864}\left(\bar{\sigma}^k\right)^{\dot{\alpha}\beta}\theta^4(D_\beta\Phi)\partial_k(D^2\Phi)(\bar{D}_{\dot{\alpha}}\Phi^+) \\
& +\frac{g^3m(288\Lambda^2\mathbf{a}_1+g^2(4(41\Lambda^2+84\sigma\Lambda\Lambda)\mathbf{a}_2-g^2(533\Lambda^2+896\sigma\Lambda\Lambda)\mathbf{a}_3))}{9216}\epsilon^{\alpha\beta}\theta^4(D_\beta\Phi)\square(D_\alpha\Phi)\Phi^+, \\
f_{18} &= \frac{g^2m^2(144\mathbf{a}_1-220g^2\mathbf{a}_2+401g^4\mathbf{a}_3)}{4608}\Lambda^{kl}\theta^4\Phi\partial_l\partial_k(D^2\Phi) \\
& +\frac{g^2m^2(144\mathbf{a}_1-220g^2\mathbf{a}_2+401g^4\mathbf{a}_3)}{4608}\epsilon^{\alpha\beta}\Lambda^{kl}\theta^4(D_\beta\Phi)\partial_l\partial_k(D_\alpha\Phi), \\
f_{19} &= \frac{1}{576}(-72g^4\mathbf{a}_1+116g^6\mathbf{a}_2-235g^8\mathbf{a}_3)\epsilon^{\alpha\beta}\left(\eta\sigma\Lambda\Lambda^k\right)^l\theta^4(D_\alpha\Phi)(D_\beta\Phi)\Phi^+\partial_k\partial_l\Phi^+ \\
& +\frac{1}{576}(72g^4\mathbf{a}_1-116g^6\mathbf{a}_2+235g^8\mathbf{a}_3)\left(\eta\sigma\Lambda\Lambda^k\right)^l\theta^4\Phi(D^2\Phi)\Phi^+\partial_k\partial_l\Phi^+, \\
f_{20} &= \frac{-72g^4\mathbf{a}_1+116g^6\mathbf{a}_2-235g^8\mathbf{a}_3}{1152}\epsilon^{\alpha\beta}\Lambda^{kl}\theta^4(D_\alpha\Phi)(D_\beta\Phi)\Phi^+\partial_k\partial_l\Phi^+
\end{aligned}$$

$$\begin{aligned}
& + \frac{72g^4 a_1 - 116g^6 a_2 + 235g^8 a_3}{1152} \Lambda^{kl} \theta^4 \Phi (D^2 \Phi) \Phi^+ \partial_k \partial_l \Phi^+, \\
f_{21} = & \frac{g^6 a_2}{288} \epsilon^{\alpha\beta} \left( \eta \sigma \Lambda^k \right)^\zeta \Lambda^l{}_\beta \theta^4 (D_\alpha \Phi) \partial_k \partial_l (D_\zeta \Phi) \Phi^+ \Phi^+ \\
& - \frac{1}{36} g^6 a_2 \epsilon^{\alpha\beta} \left( \eta \sigma \Lambda^k \right)^\zeta \Lambda^l{}_\beta \theta^4 \partial_k (D_\alpha \Phi) \partial_l (D_\zeta \Phi) \Phi^+ \Phi^+ \\
& - \frac{1}{768} i g^6 (3\Lambda^2 + 2\sigma\Lambda\Lambda) (-5a_2 + 18g^2 a_3) (\bar{\sigma}^k)^{\dot{\alpha}\beta} \theta^4 (D_\beta \Phi) (D^2 \Phi) \Phi^+ \partial_k (\bar{D}_{\dot{\alpha}} \Phi^+) \\
& + \frac{1}{576} (-72g^4 a_1 + 92g^6 a_2 - 189g^8 a_3) \epsilon^{\alpha\beta} \eta_{kl} \left( \sigma \Lambda^{ln} \right)^{o\zeta} \Lambda^k{}_\beta \theta^4 \partial_n (D_\alpha \Phi) \partial_o (D_\zeta \Phi) \Phi^+ \Phi^+ \\
& + \frac{-36g^4 a_1 + 92g^6 a_2 - 167g^8 a_3}{1152} \Lambda^{kl} \theta^4 \partial_l \Phi \partial_k (D^2 \Phi) \Phi^+ \Phi^+ \\
& + \frac{76g^6 a_2 - 145g^8 a_3}{2304} \left( \eta \sigma \Lambda \Lambda^k \right)^l \theta^4 \partial_l \Phi \partial_k (D^2 \Phi) \Phi^+ \Phi^+ \\
& + \frac{1}{288} (-36g^4 a_1 + 57g^6 a_2 - 106g^8 a_3) \epsilon^{\alpha\beta} \eta_{kl} \left( \sigma \Lambda^{ln} \right)^{o\zeta} \Lambda^k{}_\beta \theta^4 (D_\alpha \Phi) \partial_n \partial_o (D_\zeta \Phi) \Phi^+ \Phi^+ \\
& + \frac{36g^6 a_2 - 61g^8 a_3}{1152} \left( \eta \sigma \Lambda \Lambda^k \right)^l \theta^4 \partial_k \partial_l \Phi (D^2 \Phi) \Phi^+ \Phi^+ \\
& + \frac{4g^6 a_2 - 23g^8 a_3}{2304} \epsilon^{\alpha\beta} \eta_{kl} \left( \eta \sigma \Lambda^l \right)^\zeta \Lambda^k{}_\beta \theta^4 (D_\alpha \Phi) \square (D_\zeta \Phi) \Phi^+ \Phi^+ \\
& + \frac{1}{576} (6g^6 a_2 - 23g^8 a_3) \Lambda^{kl} \eta_{ln} (\sigma^{no})^{\alpha\beta} \theta^4 (D_\alpha \Phi) \partial_k \partial_o (D_\beta \Phi) \Phi^+ \Phi^+ \\
& + \left( \frac{3g^4 a_1}{32} - \frac{g^6 a_2}{9} + \frac{13g^8 a_3}{64} \right) \epsilon^{\alpha\beta} \Lambda^{kl} \theta^4 (D_\beta \Phi) \partial_l \partial_k (D_\alpha \Phi) \Phi^+ \Phi^+ \\
& + \frac{i(-4g^6 a_2 + 23g^8 a_3)}{4608} \eta_{kl} (\bar{\sigma}^l)^{\dot{\alpha}\beta} (\eta \sigma \Lambda \Lambda^n)^k \theta^4 (D_\beta \Phi) \partial_n (D^2 \Phi) \Phi^+ (\bar{D}_{\dot{\alpha}} \Phi^+) \\
& - \frac{i(-4g^6 a_2 + 23g^8 a_3)}{4608} \eta_{kl} (\bar{\sigma}^l)^{\dot{\alpha}\beta} (\eta \sigma \Lambda \Lambda^n)^k \theta^4 \partial_n (D_\beta \Phi) (D^2 \Phi) \Phi^+ (\bar{D}_{\dot{\alpha}} \Phi^+) \\
& - \frac{i(-4g^6 a_2 + 23g^8 a_3)}{4608} \epsilon^{\alpha\beta} \epsilon^{\zeta\iota} \epsilon^{kln o} \eta_{np} \eta_{oq} \Lambda^p{}_\beta \Lambda^q{}_\iota \theta^4 \partial_k (D_\alpha \Phi) \partial_l (D_\zeta \Phi) \Phi^+ \Phi^+ \\
& + \frac{28g^6 a_2 + 23g^8 a_3}{1152} \Lambda^{kl} \eta_{ln} (\sigma^{no})^{\alpha\beta} \theta^4 \partial_k (D_\alpha \Phi) \partial_o (D_\beta \Phi) \Phi^+ \Phi^+ \\
& - \frac{i(36g^4 a_1 - 52g^6 a_2 + 83g^8 a_3)}{2304} \Lambda^{kl} \eta_{ln} (\bar{\sigma}^n)^{\dot{\alpha}\beta} \theta^4 (D_\beta \Phi) \partial_k (D^2 \Phi) \Phi^+ (\bar{D}_{\dot{\alpha}} \Phi^+) \\
& + \frac{i(36g^4 a_1 - 52g^6 a_2 + 83g^8 a_3)}{2304} \Lambda^{kl} \eta_{ln} (\bar{\sigma}^n)^{\dot{\alpha}\beta} \theta^4 \partial_k (D_\beta \Phi) (D^2 \Phi) \Phi^+ (\bar{D}_{\dot{\alpha}} \Phi^+) \\
& + \frac{72g^4 a_1 - 76g^6 a_2 + 151g^8 a_3}{1152} \left( \eta \sigma \Lambda \Lambda^k \right)^l \theta^4 \Phi \partial_k \partial_l (D^2 \Phi) \Phi^+ \Phi^+ \\
& + \frac{72g^4 a_1 - 76g^6 a_2 + 151g^8 a_3}{2304} \Lambda^{kl} \theta^4 \Phi \partial_l \partial_k (D^2 \Phi) \Phi^+ \Phi^+ \\
& + \frac{1}{576} (72g^4 a_1 - 76g^6 a_2 + 151g^8 a_3) \epsilon^{\alpha\beta} \left( \eta \sigma \Lambda \Lambda^k \right)^l \theta^4 (D_\beta \Phi) \partial_k \partial_l (D_\alpha \Phi) \Phi^+ \Phi^+ \\
& + \frac{144g^4 a_1 - 140g^6 a_2 + 233g^8 a_3}{2304} \left( \eta \sigma \Lambda \Lambda^k \right)^l \theta^4 \partial_k \Phi \partial_l (D^2 \Phi) \Phi^+ \Phi^+ \\
& + \frac{144g^4 a_1 - 140g^6 a_2 + 233g^8 a_3}{1152} \epsilon^{\alpha\beta} \left( \eta \sigma \Lambda \Lambda^k \right)^l \theta^4 \partial_k (D_\beta \Phi) \partial_l (D_\alpha \Phi) \Phi^+ \Phi^+ \\
& + \frac{144g^4 a_1 - 180g^6 a_2 + 317g^8 a_3}{2304} \Lambda^{kl} \theta^4 \partial_k \partial_l \Phi (D^2 \Phi) \Phi^+ \Phi^+ \\
& + \frac{i(144g^4 a_1 - 220g^6 a_2 + 401g^8 a_3)}{4608} \eta_{kl} (\bar{\sigma}^l)^{\dot{\alpha}\beta} \left( \eta \sigma \Lambda \Lambda^k \right)^n \theta^4 (D_\beta \Phi) \partial_n (D^2 \Phi) \Phi^+ (\bar{D}_{\dot{\alpha}} \Phi^+) \\
& - \frac{i(144g^4 a_1 - 220g^6 a_2 + 401g^8 a_3)}{4608} \eta_{kl} (\bar{\sigma}^l)^{\dot{\alpha}\beta} \left( \eta \sigma \Lambda \Lambda^k \right)^n \theta^4 \partial_n (D_\beta \Phi) (D^2 \Phi) \Phi^+ (\bar{D}_{\dot{\alpha}} \Phi^+) \\
& + \frac{144g^4 a_1 - 220g^6 a_2 + 401g^8 a_3}{2304} \epsilon^{\alpha\beta} \epsilon^{\zeta\iota} \Lambda^k{}_\beta \Lambda^l{}_\iota \theta^4 \partial_k (D_\alpha \Phi) \partial_l (D_\zeta \Phi) \Phi^+ \Phi^+ \\
& + \frac{288g^4 a_1 - 588g^6 a_2 + 1069g^8 a_3}{4608} \epsilon^{\alpha\beta} \Lambda^{kl} \theta^4 \partial_k (D_\alpha \Phi) \partial_l (D_\beta \Phi) \Phi^+ \Phi^+ \\
& + \frac{g^6 ((-68\Lambda^2 + 152\sigma\Lambda\Lambda) a_2 + g^2 (521\Lambda^2 - 542\sigma\Lambda\Lambda) a_3)}{9216} \eta^{kl} \epsilon^{\alpha\beta} \theta^4 \partial_k (D_\alpha \Phi) \partial_l (D_\beta \Phi) \Phi^+ \Phi^+ \\
& + \frac{g^4 (144\Lambda^2 a_1 - 36g^2 (5\Lambda^2 - 2\sigma\Lambda\Lambda) a_2 + g^4 (673\Lambda^2 - 418\sigma\Lambda\Lambda) a_3)}{9216} \theta^4 \square \Phi (D^2 \Phi) \Phi^+ \Phi^+ \\
& + \frac{g^4 (16\Lambda^2 a_1 - 4g^2 (5\Lambda^2 - 2\sigma\Lambda\Lambda) a_2 + 5g^4 (11\Lambda^2 - 6\sigma\Lambda\Lambda) a_3)}{8192} \epsilon^{\dot{\alpha}\dot{\beta}} \theta^4 (D^2 \Phi) (D^2 \Phi) (\bar{D}_{\dot{\alpha}} \Phi^+) (\bar{D}_{\dot{\beta}} \Phi^+)
\end{aligned}$$

$$\begin{aligned}
& + \frac{g^4 (48\Lambda^2 \mathbf{a}_1 - 8g^2 (15\Lambda^2 + 2\sigma\Lambda\Lambda) \mathbf{a}_2 + 3g^4 (127\Lambda^2 + 18\sigma\Lambda\Lambda) \mathbf{a}_3)}{24576} \theta^4 (D^2\Phi) (D^2\Phi) \Phi^+ (\bar{D}^2\Phi^+) \\
& + \frac{g^4 (-36\Lambda^2 \mathbf{a}_1 - 2g^2 (\Lambda^2 + 20\sigma\Lambda\Lambda) \mathbf{a}_2 + 5g^4 (5\Lambda^2 + 19\sigma\Lambda\Lambda) \mathbf{a}_3)}{1152} \epsilon^{\alpha\beta} \theta^4 (D_\alpha\Phi) (D_\beta\Phi) \Phi^+ \square \Phi^+ \\
& + \frac{g^6 (4 (37\Lambda^2 + 4\sigma\Lambda\Lambda) \mathbf{a}_2 - g^2 (333\Lambda^2 + 136\sigma\Lambda\Lambda) \mathbf{a}_3)}{9216} \epsilon^{\alpha\beta} \theta^4 (D_\beta\Phi) \square (D_\alpha\Phi) \Phi^+ \Phi^+ \\
& + \frac{148g^6 \Lambda^2 \mathbf{a}_2 + g^8 (-689\Lambda^2 + 252\sigma\Lambda\Lambda) \mathbf{a}_3}{4608} (\sigma^{kl})^{\alpha\beta} \theta^4 \partial_k (D_\alpha\Phi) \partial_l (D_\beta\Phi) \Phi^+ \Phi^+ \\
& + \frac{i (288g^4 \Lambda^2 \mathbf{a}_1 + g^6 ((-508\Lambda^2 + 136\sigma\Lambda\Lambda) \mathbf{a}_2 + g^2 (1679\Lambda^2 - 746\sigma\Lambda\Lambda) \mathbf{a}_3))}{18432} (\bar{\sigma}^k)^{\dot{\alpha}\beta} \theta^4 \partial_k (D_\beta\Phi) (D^2\Phi) \Phi^+ (\bar{D}_{\dot{\alpha}}\Phi^+) \\
& + \frac{i (288g^4 \Lambda^2 \mathbf{a}_1 + g^6 (-4 (53\Lambda^2 - 38\sigma\Lambda\Lambda) \mathbf{a}_2 + g^2 (301\Lambda^2 - 334\sigma\Lambda\Lambda) \mathbf{a}_3))}{18432} (\bar{\sigma}^k)^{\dot{\alpha}\beta} \theta^4 (D_\beta\Phi) \partial_k (D^2\Phi) \Phi^+ (\bar{D}_{\dot{\alpha}}\Phi^+) \\
& + \frac{288g^4 \Lambda^2 \mathbf{a}_1 + g^6 (-4 (73\Lambda^2 + 2\sigma\Lambda\Lambda) \mathbf{a}_2 + g^2 (469\Lambda^2 + 2\sigma\Lambda\Lambda) \mathbf{a}_3)}{9216} \eta^{kl} \theta^4 \partial_l \Phi \partial_k (D^2\Phi) \Phi^+ \Phi^+ \\
& + \frac{36g^4 \Lambda^2 \mathbf{a}_1 + g^6 ((-43\Lambda^2 + 10\sigma\Lambda\Lambda) \mathbf{a}_2 + g^2 (137\Lambda^2 + 13\sigma\Lambda\Lambda) \mathbf{a}_3)}{1152} \theta^4 \Phi (D^2\Phi) \Phi^+ \square \Phi^+ \\
& + \frac{36g^4 \Lambda^2 \mathbf{a}_1 - g^6 (4 (2\Lambda^2 - 5\sigma\Lambda\Lambda) \mathbf{a}_2 + g^2 (4\Lambda^2 + 53\sigma\Lambda\Lambda) \mathbf{a}_3)}{2304} \theta^4 \Phi \square (D^2\Phi) \Phi^+ \Phi^+, \\
f_{22} &= \frac{g^4 m^2 (20 (\Lambda^2 + 2\sigma\Lambda\Lambda) \mathbf{a}_2 + g^2 (47\Lambda^2 - 158\sigma\Lambda\Lambda) \mathbf{a}_3)}{9216} \theta^4 \Phi \square (D^2\Phi) \\
& + \frac{g^4 m^2 (20 (\Lambda^2 + 2\sigma\Lambda\Lambda) \mathbf{a}_2 + g^2 (47\Lambda^2 - 158\sigma\Lambda\Lambda) \mathbf{a}_3)}{9216} \epsilon^{\alpha\beta} \theta^4 (D_\beta\Phi) \square (D_\alpha\Phi), \\
f_{23} &= \frac{1}{768} i g^4 m^3 (12\Lambda^2 \mathbf{a}_2 - g^2 (69\Lambda^2 + 14\sigma\Lambda\Lambda) \mathbf{a}_3) (\bar{\sigma}^k)^{\dot{\alpha}\beta} \theta^4 (D_\beta\Phi) \partial_k (\bar{D}_{\dot{\alpha}}\Phi^+) \\
& + \frac{1}{384} g^4 m^3 (-12\Lambda^2 \mathbf{a}_2 + g^2 (69\Lambda^2 + 14\sigma\Lambda\Lambda) \mathbf{a}_3) \theta^4 \Phi \square \Phi^+ \\
& + \frac{g^4 m^3 (-12\Lambda^2 \mathbf{a}_2 + g^2 (69\Lambda^2 + 14\sigma\Lambda\Lambda) \mathbf{a}_3)}{6144} \theta^4 (D^2\Phi) (\bar{D}^2\Phi^+), \\
f_{24} &= \frac{1}{128} i g^5 m^2 (12\Lambda^2 \mathbf{a}_2 - g^2 (69\Lambda^2 + 14\sigma\Lambda\Lambda) \mathbf{a}_3) (\bar{\sigma}^k)^{\dot{\alpha}\beta} \theta^4 (D_\beta\Phi) \Phi^+ \partial_k (\bar{D}_{\dot{\alpha}}\Phi^+) \\
& + \frac{1}{64} g^5 m^2 (-12\Lambda^2 \mathbf{a}_2 + g^2 (69\Lambda^2 + 14\sigma\Lambda\Lambda) \mathbf{a}_3) \theta^4 \Phi \Phi^+ \square \Phi^+ \\
& + \frac{g^5 m^2 (-12\Lambda^2 \mathbf{a}_2 + g^2 (69\Lambda^2 + 14\sigma\Lambda\Lambda) \mathbf{a}_3)}{1024} \theta^4 (D^2\Phi) \Phi^+ (\bar{D}^2\Phi^+), \\
f_{25} &= \frac{1}{64} i g^6 m (12\Lambda^2 \mathbf{a}_2 - g^2 (69\Lambda^2 + 14\sigma\Lambda\Lambda) \mathbf{a}_3) (\bar{\sigma}^k)^{\dot{\alpha}\beta} \theta^4 (D_\beta\Phi) \Phi^+ \Phi^+ \partial_k (\bar{D}_{\dot{\alpha}}\Phi^+) \\
& + \frac{1}{32} g^6 m (-12\Lambda^2 \mathbf{a}_2 + g^2 (69\Lambda^2 + 14\sigma\Lambda\Lambda) \mathbf{a}_3) \theta^4 \Phi \Phi^+ \Phi^+ \square \Phi^+ \\
& + \frac{1}{512} g^6 m (-12\Lambda^2 \mathbf{a}_2 + g^2 (69\Lambda^2 + 14\sigma\Lambda\Lambda) \mathbf{a}_3) \theta^4 (D^2\Phi) \Phi^+ \Phi^+ (\bar{D}^2\Phi^+), \\
f_{26} &= \frac{1}{96} i g^7 (12\Lambda^2 \mathbf{a}_2 - g^2 (69\Lambda^2 + 14\sigma\Lambda\Lambda) \mathbf{a}_3) (\bar{\sigma}^k)^{\dot{\alpha}\beta} \theta^4 (D_\beta\Phi) \Phi^+ \Phi^+ \Phi^+ \partial_k (\bar{D}_{\dot{\alpha}}\Phi^+) \\
& + \frac{1}{768} g^7 (-12\Lambda^2 \mathbf{a}_2 + g^2 (69\Lambda^2 + 14\sigma\Lambda\Lambda) \mathbf{a}_3) \theta^4 (D^2\Phi) \Phi^+ \Phi^+ \Phi^+ (\bar{D}^2\Phi^+) \\
& + \left( -\frac{1}{4} g^7 \Lambda^2 \mathbf{a}_2 + \frac{1}{48} g^9 (69\Lambda^2 + 14\sigma\Lambda\Lambda) \mathbf{a}_3 \right) \theta^4 \Phi \Phi^+ \Phi^+ \Phi^+ \square \Phi^+, \\
f_{27} &= \frac{g^5 (-72 (4\Lambda^2 - \sigma\Lambda\Lambda) \mathbf{a}_2 + g^2 (287\Lambda^2 - 398\sigma\Lambda\Lambda) \mathbf{a}_3)}{110592} \theta^4 \Phi (D^2\Phi) \square (D^2\Phi) \\
& + \frac{g^5 (-72\Lambda^2 \mathbf{a}_2 + g^2 (107\Lambda^2 - 74\sigma\Lambda\Lambda) \mathbf{a}_3)}{36864} \eta^{kl} \theta^4 \Phi \partial_k (D^2\Phi) \partial_l (D^2\Phi) \\
& - \frac{g^5 (36 (\Lambda^2 - \sigma\Lambda\Lambda) \mathbf{a}_2 + g^2 (17\Lambda^2 + 88\sigma\Lambda\Lambda) \mathbf{a}_3)}{27648} \epsilon^{\alpha\beta} \theta^4 (D_\beta\Phi) \square (D_\alpha\Phi) (D^2\Phi) \\
& + \frac{g^5 (-72 (\Lambda^2 + 2\sigma\Lambda\Lambda) \mathbf{a}_2 + g^2 (389\Lambda^2 + 130\sigma\Lambda\Lambda) \mathbf{a}_3)}{110592} \eta^{kl} \epsilon^{\alpha\beta} \theta^4 \partial_k (D_\alpha\Phi) \partial_l (D_\beta\Phi) (D^2\Phi), \\
f_{28} &= \frac{9g^5 \mathbf{a}_2 - 13g^7 \mathbf{a}_3}{2304} (\eta\sigma\Lambda\Lambda^k)^l \theta^4 \Phi \partial_k (D^2\Phi) \partial_l (D^2\Phi) \\
& - \frac{3}{256} (-2g^5 \mathbf{a}_2 + 3g^7 \mathbf{a}_3) \epsilon^{\alpha\beta} \eta_{kl} (\sigma\Lambda^{ln})^{\alpha\zeta} \Lambda^k{}_\beta \theta^4 \partial_n (D_\alpha\Phi) \partial_o (D_\zeta\Phi) (D^2\Phi) \\
& + \frac{5 (-18g^5 \mathbf{a}_2 + 23g^7 \mathbf{a}_3)}{6912} \epsilon^{\alpha\beta} (\eta\sigma\Lambda\Lambda^k)^l \theta^4 (D_\beta\Phi) \partial_k \partial_l (D_\alpha\Phi) (D^2\Phi) \\
& + \frac{-36g^5 \mathbf{a}_2 + 37g^7 \mathbf{a}_3}{13824} (\eta\sigma\Lambda\Lambda^k)^l \theta^4 \Phi (D^2\Phi) \partial_k \partial_l (D^2\Phi) \\
& + \frac{-396g^5 \mathbf{a}_2 + 551g^7 \mathbf{a}_3}{13824} \epsilon^{\alpha\beta} (\eta\sigma\Lambda\Lambda^k)^l \theta^4 \partial_k (D_\beta\Phi) \partial_l (D_\alpha\Phi) (D^2\Phi),
\end{aligned}$$

$$\begin{aligned}
f_{29} &= \frac{5(-18g^5 a_2 + 23g^7 a_3)}{13824} \epsilon^{\alpha\beta} \Lambda^{kl} \theta^4 (D_\beta \Phi) \partial_l \partial_k (D_\alpha \Phi) (D^2 \Phi) \\
&+ \frac{-192g a_0 - 36g^5 a_2 + 55g^7 a_3}{3072} \epsilon^{\alpha\beta} \epsilon^{\zeta\iota} \Lambda^k{}_\beta \Lambda^l{}_\iota \theta^4 \partial_k (D_\alpha \Phi) \partial_l (D_\zeta \Phi) (D^2 \Phi) \\
&+ \frac{-144g a_0 - 45g^5 a_2 + 68g^7 a_3}{4608} \Lambda^{kl} \theta^4 \Phi \partial_k (D^2 \Phi) \partial_l (D^2 \Phi) \\
&+ \frac{-864g a_0 - 360g^5 a_2 + 523g^7 a_3}{27648} \Lambda^{kl} \theta^4 \Phi (D^2 \Phi) \partial_l \partial_k (D^2 \Phi) \\
&+ \frac{-3456g a_0 - 504g^5 a_2 + 851g^7 a_3}{55296} \epsilon^{\alpha\beta} \Lambda^{kl} \theta^4 \partial_k (D_\alpha \Phi) \partial_l (D_\beta \Phi) (D^2 \Phi), \\
f_{30} &= \frac{g^3 m^5 (3\Lambda^2 + 2\sigma\Lambda\Lambda) (-10a_2 + 71g^2 a_3)}{2048} \theta^4 (D^2 \Phi), \\
f_{31} &= -\frac{g a_0}{3} (\sigma\Lambda\Lambda^{kl})^{no} \theta^4 \Phi^+ \partial_k \partial_n \Phi^+ \partial_l \partial_o \Phi^+, \\
f_{32} &= -\frac{37g^7 \Lambda^2 a_3}{6144} (\sigma^{kl})^{\alpha\beta} \theta^4 \partial_k (D_\alpha \Phi) \partial_l (D_\beta \Phi) (D^2 \Phi), \\
f_{33} &= \frac{7g^7 a_3}{54} \eta^{kl} (\eta\sigma\Lambda\Lambda^n)^o \theta^4 \Phi^+ \partial_l \partial_n \Phi^+ \partial_k \partial_o \Phi^+, \\
f_{34} &= \frac{1}{192} g^8 m a_3 (\eta\sigma\Lambda\Lambda^k)^l \theta^4 \Phi^+ \Phi^+ \partial_k \Phi^+ \partial_l \Phi^+, \\
f_{35} &= \frac{1}{576} g^8 m a_3 (\eta\sigma\Lambda\Lambda^k)^l \theta^4 \Phi^+ \Phi^+ \Phi^+ \partial_k \partial_l \Phi^+, \\
f_{36} &= \frac{g^7 a_3}{1536} \Lambda^{kl} \eta_{ln} (\sigma^{no})^{\alpha\beta} \theta^4 \partial_k (D_\alpha \Phi) \partial_o (D_\beta \Phi) (D^2 \Phi), \\
f_{37} &= -\frac{ig^7 a_3}{6144} \epsilon^{\alpha\beta} \epsilon^{\zeta\iota} \epsilon^{klno} \eta_{np} \eta_{oq} \Lambda^p{}_\beta \Lambda^q{}_\iota \theta^4 \partial_k (D_\alpha \Phi) \partial_l (D_\zeta \Phi) (D^2 \Phi), \\
f_{38} &= \frac{g^4 m^3 (3\Lambda^2 + 2\sigma\Lambda\Lambda) (-2a_2 + 15g^2 a_3)}{2048} \theta^4 (D^2 \Phi) (D^2 \Phi), \\
f_{39} &= \frac{1}{128} g^4 m^3 (3\Lambda^2 + 2\sigma\Lambda\Lambda) (-2a_2 + 15g^2 a_3) \theta^4 \Phi^+ \square \Phi^+, \\
f_{40} &= \frac{g^5 m^2 (3\Lambda^2 + 2\sigma\Lambda\Lambda) (-2a_2 + 15g^2 a_3)}{1024} \theta^4 (D^2 \Phi) (D^2 \Phi) \Phi^+, \\
f_{41} &= \frac{1}{64} g^5 m^2 (3\Lambda^2 + 2\sigma\Lambda\Lambda) (-2a_2 + 15g^2 a_3) \theta^4 \Phi^+ \Phi^+ \square \Phi^+, \\
f_{42} &= \frac{5g^4 m (-18a_2 + 23g^2 a_3)}{27648} (\eta\sigma\Lambda\Lambda^k)^l \theta^4 (D^2 \Phi) \partial_k \partial_l (D^2 \Phi), \\
f_{43} &= \frac{5g^4 m (-18a_2 + 23g^2 a_3)}{1728} (\eta\sigma\Lambda\Lambda^k)^l \theta^4 \Phi^+ \square \partial_k \partial_l \Phi^+, \\
f_{44} &= \frac{5g^4 m (-18a_2 + 23g^2 a_3)}{55296} \Lambda^{kl} \theta^4 (D^2 \Phi) \partial_l \partial_k (D^2 \Phi), \\
f_{45} &= \frac{5g^4 m (-18a_2 + 23g^2 a_3)}{3456} \Lambda^{kl} \theta^4 \Phi^+ \square \partial_k \partial_l \Phi^+, \\
f_{46} &= \frac{g^5 \Lambda^2 (9a_2 + 32g^2 a_3)}{1728} \eta^{kl} \eta^{no} \theta^4 \Phi^+ \partial_k \partial_n \Phi^+ \partial_l \partial_o \Phi^+, \\
f_{47} &= \frac{1}{96} g^6 m (-21a_2 + 47g^2 a_3) \Lambda^{kl} \theta^4 \Phi^+ \Phi^+ \partial_k \Phi^+ \partial_l \Phi^+, \\
f_{48} &= \frac{1}{288} g^6 m (-21a_2 + 47g^2 a_3) \Lambda^{kl} \theta^4 \Phi^+ \Phi^+ \Phi^+ \partial_k \partial_l \Phi^+, \\
f_{49} &= \frac{3g^4 m^4 (3\Lambda^2 + 2\sigma\Lambda\Lambda) (-10a_2 + 71g^2 a_3)}{1024} \theta^4 (D^2 \Phi) \Phi^+, \\
f_{50} &= \frac{3}{512} g^5 m^3 (3\Lambda^2 + 2\sigma\Lambda\Lambda) (-10a_2 + 71g^2 a_3) \theta^4 (D^2 \Phi) \Phi^+ \Phi^+, \\
f_{51} &= \frac{1}{256} g^6 m^2 (3\Lambda^2 + 2\sigma\Lambda\Lambda) (-10a_2 + 71g^2 a_3) \theta^4 (D^2 \Phi) \Phi^+ \Phi^+ \Phi^+, \\
f_{52} &= \frac{1}{864} g^5 (-54a_2 + 143g^2 a_3) (\eta\sigma\Lambda\Lambda^k)^l \theta^4 \Phi^+ \square \Phi^+ \partial_k \partial_l \Phi^+, \\
f_{53} &= \frac{g^3 m (72a_1 - 76g^2 a_2 + 151g^4 a_3)}{1152} (\eta\sigma\Lambda\Lambda^k)^l \theta^4 (D^2 \Phi) \partial_k \Phi^+ \partial_l \Phi^+, \\
f_{54} &= \frac{g^3 m (72a_1 - 76g^2 a_2 + 151g^4 a_3)}{2304} \Lambda^{kl} \theta^4 (D^2 \Phi) \partial_k \Phi^+ \partial_l \Phi^+, \\
f_{55} &= \frac{g^2 m^2 (72a_1 - 96g^2 a_2 + 193g^4 a_3)}{1152} (\eta\sigma\Lambda\Lambda^k)^l \theta^4 (D^2 \Phi) \partial_k \partial_l \Phi^+, \\
f_{56} &= \frac{g^2 m^2 (72a_1 - 96g^2 a_2 + 193g^4 a_3)}{2304} \Lambda^{kl} \theta^4 (D^2 \Phi) \partial_k \partial_l \Phi^+,
\end{aligned}$$



$$\begin{aligned}
f_{57} &= \frac{1}{576} g^3 m (72 \mathbf{a}_1 - 96 g^2 \mathbf{a}_2 + 193 g^4 \mathbf{a}_3) \left( \eta \sigma \Lambda \Lambda^k \right)^l \theta^4 (D^2 \Phi) \Phi^+ \partial_k \partial_l \Phi^+, \\
f_{58} &= \frac{g^3 m (72 \mathbf{a}_1 - 96 g^2 \mathbf{a}_2 + 193 g^4 \mathbf{a}_3)}{1152} \Lambda^{kl} \theta^4 (D^2 \Phi) \Phi^+ \partial_k \partial_l \Phi^+, \\
f_{59} &= \frac{1}{192} (-2 g^2 \mathbf{a}_1 + g^6 \mathbf{a}_3) \left( \eta \sigma \Lambda \Lambda^k \right)^l \theta^4 (D^2 \Phi) \square \partial_k \partial_l \Phi^+, \\
f_{60} &= \frac{1}{384} (-2 g^2 \mathbf{a}_1 + g^6 \mathbf{a}_3) \Lambda^{kl} \theta^4 (D^2 \Phi) \square \partial_k \partial_l \Phi^+, \\
f_{61} &= \frac{1}{432} (-9 g^5 \mathbf{a}_2 - 35 g^7 \mathbf{a}_3) \left( \eta \sigma \Lambda \Lambda^k \right)^l \theta^4 \Phi^+ \Phi^+ \square \partial_k \partial_l \Phi^+, \\
f_{62} &= \left( \frac{g \mathbf{a}_0}{6} + \frac{g^5 \mathbf{a}_2}{96} - \frac{2 g^7 \mathbf{a}_3}{27} \right) \eta^{kl} \Lambda^{no} \theta^4 \Phi^+ \partial_k \partial_n \Phi^+ \partial_l \partial_o \Phi^+, \\
f_{63} &= \left( -\frac{7}{384} g^5 \mathbf{a}_2 + \frac{55 g^7 \mathbf{a}_3}{864} \right) \Lambda^{kl} \theta^4 \Phi^+ \Phi^+ \square \partial_k \partial_l \Phi^+, \\
f_{64} &= \frac{-288 g \mathbf{a}_0 - 45 g^5 \mathbf{a}_2 + 23 g^7 \mathbf{a}_3}{1728} \Lambda^{kl} \theta^4 \Phi^+ \square \Phi^+ \partial_k \partial_l \Phi^+, \\
f_{65} &= \frac{g^5 (-45 \Lambda^2 \mathbf{a}_2 + g^2 (29 \Lambda^2 - 60 \sigma \Lambda \Lambda) \mathbf{a}_3)}{3456} \theta^4 \Phi^+ \square \Phi^+ \square \Phi^+, \\
f_{66} &= \frac{1}{384} g^6 m (2 (53 \Lambda^2 - 12 \sigma \Lambda \Lambda) \mathbf{a}_2 + g^2 (68 \Lambda^2 - 37 \sigma \Lambda \Lambda) \mathbf{a}_3) \eta^{kl} \theta^4 \Phi^+ \Phi^+ \partial_k \Phi^+ \partial_l \Phi^+, \\
f_{67} &= -\frac{g^2 (4 \Lambda^2 \mathbf{a}_1 + g^4 (\Lambda^2 + 2 \sigma \Lambda \Lambda) \mathbf{a}_3)}{1536} \theta^4 (D^2 \Phi) \square \square \Phi^+, \\
f_{68} &= -\frac{g^5 (9 (\Lambda^2 - 4 \sigma \Lambda \Lambda) \mathbf{a}_2 + 2 g^2 (55 \Lambda^2 + 14 \sigma \Lambda \Lambda) \mathbf{a}_3)}{6912} \theta^4 \Phi^+ \Phi^+ \square \square \Phi^+, \\
f_{69} &= \frac{g^6 m ((34 \Lambda^2 - 72 \sigma \Lambda \Lambda) \mathbf{a}_2 + 19 g^2 (32 \Lambda^2 + 17 \sigma \Lambda \Lambda) \mathbf{a}_3)}{1152} \theta^4 \Phi^+ \Phi^+ \Phi^+ \square \Phi^+, \\
f_{70} &= \frac{g^3 m (4 (9 \Lambda^2 \mathbf{a}_1 + g^2 (-2 \Lambda^2 + 5 \sigma \Lambda \Lambda) \mathbf{a}_2) - g^4 (4 \Lambda^2 + 53 \sigma \Lambda \Lambda) \mathbf{a}_3)}{2304} \eta^{kl} \theta^4 (D^2 \Phi) \partial_k \Phi^+ \partial_l \Phi^+, \\
f_{71} &= -\frac{g^4 m (36 (\Lambda^2 - \sigma \Lambda \Lambda) \mathbf{a}_2 + g^2 (17 \Lambda^2 + 88 \sigma \Lambda \Lambda) \mathbf{a}_3)}{110592} \theta^4 (D^2 \Phi) \square (D^2 \Phi), \\
f_{72} &= -\frac{g^4 m (36 (\Lambda^2 - \sigma \Lambda \Lambda) \mathbf{a}_2 + g^2 (17 \Lambda^2 + 88 \sigma \Lambda \Lambda) \mathbf{a}_3)}{6912} \theta^4 \Phi^+ \square \square \Phi^+, \\
f_{73} &= \frac{g^2 m^2 (72 \Lambda^2 \mathbf{a}_1 + g^2 ((-51 \Lambda^2 + 30 \sigma \Lambda \Lambda) \mathbf{a}_2 + g^2 (133 \Lambda^2 - 40 \sigma \Lambda \Lambda) \mathbf{a}_3))}{4608} \theta^4 (D^2 \Phi) \square \Phi^+, \\
f_{74} &= \frac{g^3 m (72 \Lambda^2 \mathbf{a}_1 + g^2 ((-51 \Lambda^2 + 30 \sigma \Lambda \Lambda) \mathbf{a}_2 + g^2 (133 \Lambda^2 - 40 \sigma \Lambda \Lambda) \mathbf{a}_3))}{2304} \theta^4 (D^2 \Phi) \Phi^+ \square \Phi^+. \tag{B1}
\end{aligned}$$

## B.2 Bases

We list the 74 bases  $B_i$ 's below, each of them is invariant under the  $1/2$  supersymmetry transformation..

$$\begin{aligned}
B_1 &= (-\Lambda^2 y_{1,2} - \sigma \Lambda \Lambda z_{1,2}) \epsilon^{\alpha\beta} \theta^4 (D_\alpha \Phi) (D_\beta \Phi) \\
&\quad + (\Lambda^2 y_{1,2} + \sigma \Lambda \Lambda z_{1,2}) \theta^4 \Phi (D^2 \Phi), \\
B_2 &= (-2 \Lambda^2 y_{2,2} - 2 \sigma \Lambda \Lambda z_{2,2}) \epsilon^{\alpha\beta} \theta^4 \Phi (D_\alpha \Phi) (D_\beta \Phi) \\
&\quad + (\Lambda^2 y_{2,2} + \sigma \Lambda \Lambda z_{2,2}) \theta^4 \Phi \Phi (D^2 \Phi), \\
B_3 &= (-\Lambda^2 y_{3,2} - \sigma \Lambda \Lambda z_{3,2}) \epsilon^{\alpha\beta} \theta^4 (D_\alpha \Phi) (D_\beta \Phi) \Phi^+ \\
&\quad + (\Lambda^2 y_{3,2} + \sigma \Lambda \Lambda z_{3,2}) \theta^4 \Phi (D^2 \Phi) \Phi^+, \\
B_4 &= (-2 \Lambda^2 y_{4,2} - 2 \sigma \Lambda \Lambda z_{4,2}) \epsilon^{\alpha\beta} \theta^4 \Phi (D_\alpha \Phi) (D_\beta \Phi) \Phi^+ \\
&\quad + (\Lambda^2 y_{4,2} + \sigma \Lambda \Lambda z_{4,2}) \theta^4 \Phi \Phi (D^2 \Phi) \Phi^+, \\
B_5 &= (-\Lambda^2 y_{5,2} - \sigma \Lambda \Lambda z_{5,2}) \epsilon^{\alpha\beta} \theta^4 (D_\alpha \Phi) (D_\beta \Phi) \Phi^+ \Phi^+ \\
&\quad + (\Lambda^2 y_{5,2} + \sigma \Lambda \Lambda z_{5,2}) \theta^4 \Phi (D^2 \Phi) \Phi^+ \Phi^+, \\
B_6 &= (-2 \Lambda^2 y_{6,2} - 2 \sigma \Lambda \Lambda z_{6,2}) \epsilon^{\alpha\beta} \theta^4 \Phi (D_\alpha \Phi) (D_\beta \Phi) \Phi^+ \Phi^+
\end{aligned}$$

$$\begin{aligned}
& + (\Lambda^2 y_{6,2} + \sigma \Lambda \Lambda z_{6,2}) \theta^4 \Phi \Phi (D^2 \Phi) \Phi^+ \Phi^+, \\
B_7 = & (-\Lambda^2 y_{7,2} - \sigma \Lambda \Lambda z_{7,2}) \epsilon^{\alpha\beta} \theta^4 (D_\alpha \Phi) (D_\beta \Phi) \Phi^+ \Phi^+ \Phi^+ \\
& + (\Lambda^2 y_{7,2} + \sigma \Lambda \Lambda z_{7,2}) \theta^4 \Phi (D^2 \Phi) \Phi^+ \Phi^+ \Phi^+, \\
B_8 = & (-2\Lambda^2 y_{8,2} - 2\sigma \Lambda \Lambda z_{8,2}) \epsilon^{\alpha\beta} \theta^4 \Phi (D_\alpha \Phi) (D_\beta \Phi) \Phi^+ \Phi^+ \Phi^+ \\
& + (\Lambda^2 y_{8,2} + \sigma \Lambda \Lambda z_{8,2}) \theta^4 \Phi \Phi (D^2 \Phi) \Phi^+ \Phi^+ \Phi^+, \\
B_9 = & (-\Lambda^2 y_{9,2} - \sigma \Lambda \Lambda z_{9,2}) \epsilon^{\alpha\beta} \theta^4 (D_\alpha \Phi) (D_\beta \Phi) (D^2 \Phi) \\
& + (\Lambda^2 y_{9,2} + \sigma \Lambda \Lambda z_{9,2}) \theta^4 \Phi (D^2 \Phi) (D^2 \Phi), \\
B_{10} = & (-2\Lambda^2 y_{10,2} - 2\sigma \Lambda \Lambda z_{10,2}) \epsilon^{\alpha\beta} \theta^4 \Phi (D_\alpha \Phi) (D_\beta \Phi) (D^2 \Phi) \\
& + (\Lambda^2 y_{10,2} + \sigma \Lambda \Lambda z_{10,2}) \theta^4 \Phi \Phi (D^2 \Phi) (D^2 \Phi), \\
B_{11} = & (-\Lambda^2 y_{11,2} - \sigma \Lambda \Lambda z_{11,2}) \epsilon^{\alpha\beta} \theta^4 (D_\alpha \Phi) (D_\beta \Phi) (D^2 \Phi) \Phi^+ \\
& + (\Lambda^2 y_{11,2} + \sigma \Lambda \Lambda z_{11,2}) \theta^4 \Phi (D^2 \Phi) (D^2 \Phi) \Phi^+, \\
B_{12} = & (-2\Lambda^2 y_{12,2} - 2\sigma \Lambda \Lambda z_{12,2}) \epsilon^{\alpha\beta} \theta^4 \Phi (D_\alpha \Phi) (D_\beta \Phi) (D^2 \Phi) \Phi^+ \\
& + (\Lambda^2 y_{12,2} + \sigma \Lambda \Lambda z_{12,2}) \theta^4 \Phi \Phi (D^2 \Phi) (D^2 \Phi) \Phi^+, \\
B_{13} = & (-8i\Lambda^2 y_{13,3} - 8i\sigma \Lambda \Lambda z_{13,3}) (\bar{\sigma}^k)^{\dot{\alpha}\beta} \theta^4 (D_\beta \Phi) \partial_k \Phi^+ (\bar{D}_{\dot{\alpha}} \Phi^+) \\
& + (16\Lambda^2 y_{13,3} + 16\sigma \Lambda \Lambda z_{13,3}) \eta^{kl} \theta^4 \Phi \partial_k \Phi^+ \partial_l \Phi^+ \\
& + (\Lambda^2 y_{13,3} + \sigma \Lambda \Lambda z_{13,3}) \epsilon^{\dot{\alpha}\dot{\beta}} \theta^4 (D^2 \Phi) (\bar{D}_{\dot{\alpha}} \Phi^+) (\bar{D}_{\dot{\beta}} \Phi^+), \\
B_{14} = & (-8i\Lambda^2 y_{14,3} - 8i\sigma \Lambda \Lambda z_{14,3}) (\bar{\sigma}^k)^{\dot{\alpha}\beta} \theta^4 (D_\beta \Phi) \Phi^+ \partial_k \Phi^+ (\bar{D}_{\dot{\alpha}} \Phi^+) \\
& + (16\Lambda^2 y_{14,3} + 16\sigma \Lambda \Lambda z_{14,3}) \eta^{kl} \theta^4 \Phi \Phi^+ \partial_k \Phi^+ \partial_l \Phi^+ \\
& + (\Lambda^2 y_{14,3} + \sigma \Lambda \Lambda z_{14,3}) \epsilon^{\dot{\alpha}\dot{\beta}} \theta^4 (D^2 \Phi) \Phi^+ (\bar{D}_{\dot{\alpha}} \Phi^+) (\bar{D}_{\dot{\beta}} \Phi^+), \\
B_{15} = & (-8i\Lambda^2 y_{15,3} - 8i\sigma \Lambda \Lambda z_{15,3}) (\bar{\sigma}^k)^{\dot{\alpha}\beta} \theta^4 (D_\beta \Phi) \Phi^+ \Phi^+ \partial_k \Phi^+ (\bar{D}_{\dot{\alpha}} \Phi^+) \\
& + (16\Lambda^2 y_{15,3} + 16\sigma \Lambda \Lambda z_{15,3}) \eta^{kl} \theta^4 \Phi \Phi^+ \Phi^+ \partial_k \Phi^+ \partial_l \Phi^+ \\
& + (\Lambda^2 y_{15,3} + \sigma \Lambda \Lambda z_{15,3}) \epsilon^{\dot{\alpha}\dot{\beta}} \theta^4 (D^2 \Phi) \Phi^+ \Phi^+ (\bar{D}_{\dot{\alpha}} \Phi^+) (\bar{D}_{\dot{\beta}} \Phi^+), \\
B_{16} = & x_{16,2} (\eta \sigma \Lambda \Lambda^k)^l \theta^4 \Phi \partial_k \partial_l (D^2 \Phi) \\
& + x_{16,2} \epsilon^{\alpha\beta} (\eta \sigma \Lambda \Lambda^k)^l \theta^4 (D_\beta \Phi) \partial_k \partial_l (D_\alpha \Phi), \\
B_{17} = & (4x_{17,16} + x_{17,21} - 2x_{17,22} + x_{17,26} + 4z_{17,30} + 8iz_{17,34} - 2z_{17,35}) \epsilon^{\alpha\beta} (\eta \sigma \Lambda^k)^\zeta \Lambda^l{}_\beta \theta^4 (D_\alpha \Phi) \partial_k \partial_l (D_\zeta \Phi) \Phi^+ \\
& + (x_{17,14} + 2x_{17,21} - 8z_{17,30} - 16iz_{17,34} + 4z_{17,35}) \epsilon^{\alpha\beta} (\eta \sigma \Lambda^k)^\zeta \Lambda^l{}_\beta \theta^4 \partial_k (D_\alpha \Phi) \partial_l (D_\zeta \Phi) \Phi^+ \\
& + \left( iz_{17,30} - 2z_{17,34} - \frac{1}{2} iz_{17,35} \right) \eta_{kl} (\bar{\sigma}^l)^{\dot{\alpha}\beta} (\eta \sigma \Lambda \Lambda^n)^k \theta^4 (D_\beta \Phi) \partial_n (D^2 \Phi) (\bar{D}_{\dot{\alpha}} \Phi^+) \\
& + \left( -iz_{17,30} + 2z_{17,34} + \frac{1}{2} iz_{17,35} \right) \eta_{kl} (\bar{\sigma}^l)^{\dot{\alpha}\beta} (\eta \sigma \Lambda \Lambda^n)^k \theta^4 \partial_n (D_\beta \Phi) (D^2 \Phi) (\bar{D}_{\dot{\alpha}} \Phi^+) \\
& + (-2iz_{17,30} + 4z_{17,34} + iz_{17,35}) \epsilon^{\alpha\beta} \epsilon^{\zeta\iota} \epsilon^{klno} \eta_{np} \eta_{oq} \Lambda^p{}_\beta \Lambda^q{}_\iota \theta^4 \partial_k (D_\alpha \Phi) \partial_l (D_\zeta \Phi) \Phi^+ \\
& + (-x_{17,14} + 4x_{17,16} - x_{17,21} - 2x_{17,22} + 4z_{17,30} + 8iz_{17,34} - 2z_{17,35}) \Lambda^{kl} \eta_{ln} (\sigma^{no})^{\alpha\beta} \theta^4 \partial_k (D_\alpha \Phi) \partial_o (D_\beta \Phi) \Phi^+ \\
& + \left( -x_{17,8} + \frac{x_{17,21}}{2} \right) \epsilon^{\alpha\beta} \Lambda^{kl} \theta^4 \partial_k (D_\alpha \Phi) \partial_l (D_\beta \Phi) \Phi^+ \\
& + x_{17,8} \Lambda^{kl} \theta^4 \partial_l \Phi \partial_k (D^2 \Phi) \Phi^+ \\
& + (-x_{17,16} + x_{17,22}) \Lambda^{kl} \theta^4 \partial_k \partial_l \Phi (D^2 \Phi) \Phi^+ \\
& + \left( \frac{1}{2} ix_{17,16} - \frac{1}{4} ix_{17,22} \right) \Lambda^{kl} \eta_{ln} (\bar{\sigma}^n)^{\dot{\alpha}\beta} \theta^4 (D_\beta \Phi) \partial_k (D^2 \Phi) (\bar{D}_{\dot{\alpha}} \Phi^+) \\
& + \left( -\frac{1}{2} ix_{17,16} + \frac{1}{4} ix_{17,22} \right) \Lambda^{kl} \eta_{ln} (\bar{\sigma}^n)^{\dot{\alpha}\beta} \theta^4 \partial_k (D_\beta \Phi) (D^2 \Phi) (\bar{D}_{\dot{\alpha}} \Phi^+) \\
& + \left( \frac{x_{17,17}}{2} - x_{17,21} - 4z_{17,30} - 8iz_{17,34} + 2z_{17,35} \right) (\eta \sigma \Lambda \Lambda^k)^l \theta^4 \partial_k \partial_l \Phi (D^2 \Phi) \Phi^+ \\
& + (4x_{17,16} - x_{17,21} - 2x_{17,22} + x_{17,26} - 4z_{17,30} - 8iz_{17,34} + 2z_{17,35}) \epsilon^{\alpha\beta} \eta_{kl} (\sigma \Lambda^{ln})^{o\zeta} \Lambda^k{}_\beta \theta^4 (D_\alpha \Phi) \partial_n \partial_o (D_\zeta \Phi) \Phi^+ \\
& + x_{17,14} \epsilon^{\alpha\beta} \eta_{kl} (\sigma \Lambda^{ln})^{o\zeta} \Lambda^k{}_\beta \theta^4 \partial_n (D_\alpha \Phi) \partial_o (D_\zeta \Phi) \Phi^+
\end{aligned}$$

$$\begin{aligned}
& +\frac{1}{2}x_{17,17}\left(\eta\sigma\Lambda\Lambda^k\right)^l\theta^4\Phi\partial_k\partial_l\left(D^2\Phi\right)\Phi^+ \\
& +x_{17,16}\Lambda^{kl}\theta^4\Phi\partial_l\partial_k\left(D^2\Phi\right)\Phi^+ \\
& +x_{17,17}\epsilon^{\alpha\beta}\left(\eta\sigma\Lambda\Lambda^k\right)^l\theta^4\left(D_\beta\Phi\right)\partial_k\partial_l\left(D_\alpha\Phi\right)\Phi^+ \\
& +\left(-x_{17,21}+\frac{x_{17,24}}{2}+4z_{17,30}+8iz_{17,34}-2z_{17,35}\right)\left(\eta\sigma\Lambda\Lambda^k\right)^l\theta^4\partial_l\Phi\partial_k\left(D^2\Phi\right)\Phi^+ \\
& +\frac{1}{4}\left(ix_{17,21}\right)\eta_{kl}\left(\bar{\sigma}^l\right)^{\dot{\alpha}\beta}\left(\eta\sigma\Lambda\Lambda^k\right)^n\theta^4\left(D_\beta\Phi\right)\partial_n\left(D^2\Phi\right)\left(\bar{D}_\alpha\Phi^+\right) \\
& +\frac{1}{4}\left(-ix_{17,21}\right)\eta_{kl}\left(\bar{\sigma}^l\right)^{\dot{\alpha}\beta}\left(\eta\sigma\Lambda\Lambda^k\right)^n\theta^4\partial_n\left(D_\beta\Phi\right)\left(D^2\Phi\right)\left(\bar{D}_\alpha\Phi^+\right) \\
& +x_{17,21}\epsilon^{\alpha\beta}\epsilon^{\zeta\iota}\Lambda^k{}_\beta\Lambda^l{}_\iota\theta^4\partial_k\left(D_\alpha\Phi\right)\partial_l\left(D_\zeta\Phi\right)\Phi^+ \\
& +x_{17,22}\epsilon^{\alpha\beta}\Lambda^{kl}\theta^4\left(D_\beta\Phi\right)\partial_l\partial_k\left(D_\alpha\Phi\right)\Phi^+ \\
& +\frac{1}{2}x_{17,24}\left(\eta\sigma\Lambda\Lambda^k\right)^l\theta^4\partial_k\Phi\partial_l\left(D^2\Phi\right)\Phi^+ \\
& +x_{17,24}\epsilon^{\alpha\beta}\left(\eta\sigma\Lambda\Lambda^k\right)^l\theta^4\partial_k\left(D_\beta\Phi\right)\partial_l\left(D_\alpha\Phi\right)\Phi^+ \\
& +\left(-4z_{17,30}-8iz_{17,34}+2z_{17,35}\right)\epsilon^{\alpha\beta}\eta_{kl}\left(\eta\sigma\Lambda^l\right)^\zeta\Lambda^k{}_\beta\theta^4\left(D_\alpha\Phi\right)\square\left(D_\zeta\Phi\right)\Phi^+ \\
& +x_{17,26}\Lambda^{kl}\eta_{ln}\left(\sigma^{no}\right)^{\alpha\beta}\theta^4\left(D_\alpha\Phi\right)\partial_k\partial_o\left(D_\beta\Phi\right)\Phi^+ \\
& +\left(\Lambda^2\left(-2iy_{17,32}+\frac{y_{17,35}}{2}\right)+\sigma\Lambda\Lambda\left(z_{17,30}-2iz_{17,32}+2iz_{17,34}\right)\right)\theta^4\square\Phi\left(D^2\Phi\right)\Phi^+ \\
& +\left(\Lambda^2\left(-\frac{1}{16}iy_{17,32}-\frac{1}{16}iy_{17,34}\right)+\sigma\Lambda\Lambda\left(-\frac{1}{16}iz_{17,32}-\frac{1}{16}iz_{17,34}\right)\right)\theta^4\left(D^2\Phi\right)\left(D^2\Phi\right)\left(\bar{D}^2\Phi^+\right) \\
& +\left(\Lambda^2\left(4iy_{17,32}-4iy_{17,34}\right)+\sigma\Lambda\Lambda\left(4z_{17,30}+4iz_{17,32}+4iz_{17,34}-2z_{17,35}\right)\right)\left(\sigma^{kl}\right)^{\alpha\beta}\theta^4\partial_k\left(D_\alpha\Phi\right)\partial_l\left(D_\beta\Phi\right)\Phi^+ \\
& +\left(\Lambda^2\left(-2iy_{17,34}+\frac{y_{17,35}}{2}\right)+\sigma\Lambda\Lambda z_{17,30}\right)\theta^4\Phi\square\left(D^2\Phi\right)\Phi^+ \\
& +\left(\Lambda^2\left(-2iy_{17,32}-y_{17,33}-2iy_{17,34}\right)+\sigma\Lambda\Lambda\left(-2iz_{17,32}-z_{17,33}-2iz_{17,34}\right)\right)\eta^{kl}\theta^4\partial_l\Phi\partial_k\left(D^2\Phi\right)\Phi^+ \\
& +\left(\Lambda^2y_{17,32}+\sigma\Lambda\Lambda z_{17,32}\right)\left(\bar{\sigma}^k\right)^{\dot{\alpha}\beta}\theta^4\partial_k\left(D_\beta\Phi\right)\left(D^2\Phi\right)\left(\bar{D}_\alpha\Phi^+\right) \\
& +\left(\Lambda^2y_{17,33}+\sigma\Lambda\Lambda z_{17,33}\right)\eta^{kl}\epsilon^{\alpha\beta}\theta^4\partial_k\left(D_\alpha\Phi\right)\partial_l\left(D_\beta\Phi\right)\Phi^+ \\
& +\left(\Lambda^2y_{17,34}+\sigma\Lambda\Lambda z_{17,34}\right)\left(\bar{\sigma}^k\right)^{\dot{\alpha}\beta}\theta^4\left(D_\beta\Phi\right)\partial_k\left(D^2\Phi\right)\left(\bar{D}_\alpha\Phi^+\right) \\
& +\left(\Lambda^2y_{17,35}+\sigma\Lambda\Lambda z_{17,35}\right)\epsilon^{\alpha\beta}\theta^4\left(D_\beta\Phi\right)\square\left(D_\alpha\Phi\right)\Phi^+,
\end{aligned}$$

$$B_{18} = x_{18,2}\Lambda^{kl}\theta^4\Phi\partial_l\partial_k\left(D^2\Phi\right)$$

$$+x_{18,2}\epsilon^{\alpha\beta}\Lambda^{kl}\theta^4\left(D_\beta\Phi\right)\partial_l\partial_k\left(D_\alpha\Phi\right),$$

$$B_{19} = -x_{19,2}\epsilon^{\alpha\beta}\left(\eta\sigma\Lambda\Lambda^k\right)^l\theta^4\left(D_\alpha\Phi\right)\left(D_\beta\Phi\right)\Phi^+\partial_k\partial_l\Phi^+$$

$$+x_{19,2}\left(\eta\sigma\Lambda\Lambda^k\right)^l\theta^4\Phi\left(D^2\Phi\right)\Phi^+\partial_k\partial_l\Phi^+,$$

$$B_{20} = -x_{20,2}\epsilon^{\alpha\beta}\Lambda^{kl}\theta^4\left(D_\alpha\Phi\right)\left(D_\beta\Phi\right)\Phi^+\partial_k\partial_l\Phi^+$$

$$+x_{20,2}\Lambda^{kl}\theta^4\Phi\left(D^2\Phi\right)\Phi^+\partial_k\partial_l\Phi^+,$$

$$B_{21} = \left(x_{21,10}+2x_{21,19}-2x_{21,23}+x_{21,26}+z_{21,34}-2iz_{21,35}+2iz_{21,36}\right)\epsilon^{\alpha\beta}\left(\eta\sigma\Lambda^k\right)^\zeta\Lambda^l{}_\beta\theta^4\left(D_\alpha\Phi\right)\partial_k\partial_l\left(D_\zeta\Phi\right)\Phi^+\Phi^+$$

$$+\left(-x_{21,15}+2x_{21,19}-2x_{21,23}+x_{21,26}-z_{21,34}+2iz_{21,35}-2iz_{21,36}\right)\epsilon^{\alpha\beta}\left(\eta\sigma\Lambda^k\right)^\zeta\Lambda^l{}_\beta\theta^4\partial_k\left(D_\alpha\Phi\right)\partial_l\left(D_\zeta\Phi\right)\Phi^+\Phi^+$$

$$+\left(\Lambda^2\left(-\frac{1}{2}iy_{21,32}-\frac{1}{2}iy_{21,38}\right)+\sigma\Lambda\Lambda\left(-\frac{1}{2}iz_{21,32}-\frac{1}{2}iz_{21,38}\right)\right)\left(\bar{\sigma}^k\right)^{\dot{\alpha}\beta}\theta^4\left(D_\beta\Phi\right)\left(D^2\Phi\right)\Phi^+\partial_k\left(\bar{D}_\alpha\Phi^+\right)$$

$$+\left(-x_{21,15}+2x_{21,19}-2x_{21,23}-x_{21,26}+z_{21,34}-2iz_{21,35}+2iz_{21,36}\right)\epsilon^{\alpha\beta}\eta_{kl}\left(\sigma\Lambda^{ln}\right)^{o\zeta}\Lambda^k{}_\beta\theta^4\partial_n\left(D_\alpha\Phi\right)\partial_o\left(D_\zeta\Phi\right)\Phi^+\Phi^+$$

$$+\left(\frac{x_{21,26}}{2}-x_{21,27}\right)\Lambda^{kl}\theta^4\partial_l\Phi\partial_k\left(D^2\Phi\right)\Phi^+\Phi^+$$

$$+\left(\frac{x_{21,22}}{2}-x_{21,26}+z_{21,34}-2iz_{21,35}+2iz_{21,36}\right)\left(\eta\sigma\Lambda\Lambda^k\right)^l\theta^4\partial_l\Phi\partial_k\left(D^2\Phi\right)\Phi^+\Phi^+$$

$$+\left(x_{21,10}+2x_{21,19}-2x_{21,23}-x_{21,26}-z_{21,34}+2iz_{21,35}-2iz_{21,36}\right)\epsilon^{\alpha\beta}\eta_{kl}\left(\sigma\Lambda^{ln}\right)^{o\zeta}\Lambda^k{}_\beta\theta^4\left(D_\alpha\Phi\right)\partial_n\partial_o\left(D_\zeta\Phi\right)\Phi^+\Phi^+$$

$$+\left(\frac{x_{21,20}}{2}-x_{21,26}-z_{21,34}+2iz_{21,35}-2iz_{21,36}\right)\left(\eta\sigma\Lambda\Lambda^k\right)^l\theta^4\partial_k\partial_l\Phi\left(D^2\Phi\right)\Phi^+\Phi^+$$

$$+\left(-z_{21,34}+2iz_{21,35}-2iz_{21,36}\right)\epsilon^{\alpha\beta}\eta_{kl}\left(\eta\sigma\Lambda^l\right)^\zeta\Lambda^k{}_\beta\theta^4\left(D_\alpha\Phi\right)\square\left(D_\zeta\Phi\right)\Phi^+\Phi^+$$

$$+x_{21,10}\Lambda^{kl}\eta_{ln}\left(\sigma^{no}\right)^{\alpha\beta}\theta^4\left(D_\alpha\Phi\right)\partial_k\partial_o\left(D_\beta\Phi\right)\Phi^+\Phi^+$$

$$\begin{aligned}
& + (x_{21,19} + x_{21,23}) \epsilon^{\alpha\beta} \Lambda^{kl} \theta^4 (D_\beta \Phi) \partial_l \partial_k (D_\alpha \Phi) \Phi^+ \Phi^+ \\
& + \left( \frac{1}{2} i z_{21,34} + z_{21,35} - z_{21,36} \right) \eta_{kl} \left( \bar{\sigma}^l \right)^{\dot{\alpha}\beta} (\eta \sigma \Lambda \Lambda^n)^k \theta^4 (D_\beta \Phi) \partial_n (D^2 \Phi) \Phi^+ (\bar{D}_{\dot{\alpha}} \Phi^+) \\
& + \left( -\frac{1}{2} i z_{21,34} - z_{21,35} + z_{21,36} \right) \eta_{kl} \left( \bar{\sigma}^l \right)^{\dot{\alpha}\beta} (\eta \sigma \Lambda \Lambda^n)^k \theta^4 \partial_n (D_\beta \Phi) (D^2 \Phi) \Phi^+ (\bar{D}_{\dot{\alpha}} \Phi^+) \\
& + \left( -\frac{1}{2} i z_{21,34} - z_{21,35} + z_{21,36} \right) \epsilon^{\alpha\beta} \epsilon^{\zeta\iota} \epsilon^{klno} \eta_{np} \eta_{oq} \Lambda^p_\beta \Lambda^q_\iota \theta^4 \partial_k (D_\alpha \Phi) \partial_l (D_\zeta \Phi) \Phi^+ \Phi^+ \\
& + x_{21,15} \Lambda^{kl} \eta_{ln} (\sigma^{no})^{\alpha\beta} \theta^4 \partial_k (D_\alpha \Phi) \partial_o (D_\beta \Phi) \Phi^+ \Phi^+ \\
& + \left( \frac{1}{2} i x_{21,19} - \frac{1}{2} i x_{21,23} \right) \Lambda^{kl} \eta_{ln} (\bar{\sigma}^n)^{\dot{\alpha}\beta} \theta^4 (D_\beta \Phi) \partial_k (D^2 \Phi) \Phi^+ (\bar{D}_{\dot{\alpha}} \Phi^+) \\
& + \left( -\frac{1}{2} i x_{21,19} + \frac{1}{2} i x_{21,23} \right) \Lambda^{kl} \eta_{ln} (\bar{\sigma}^n)^{\dot{\alpha}\beta} \theta^4 \partial_k (D_\beta \Phi) (D^2 \Phi) \Phi^+ (\bar{D}_{\dot{\alpha}} \Phi^+) \\
& + \frac{1}{2} x_{21,20} \left( \eta \sigma \Lambda \Lambda^k \right)^l \theta^4 \Phi \partial_k \partial_l (D^2 \Phi) \Phi^+ \Phi^+ \\
& + x_{21,19} \Lambda^{kl} \theta^4 \Phi \partial_l \partial_k (D^2 \Phi) \Phi^+ \Phi^+ \\
& + x_{21,20} \epsilon^{\alpha\beta} \left( \eta \sigma \Lambda \Lambda^k \right)^l \theta^4 (D_\beta \Phi) \partial_k \partial_l (D_\alpha \Phi) \Phi^+ \Phi^+ \\
& + \frac{1}{2} x_{21,22} \left( \eta \sigma \Lambda \Lambda^k \right)^l \theta^4 \partial_k \Phi \partial_l (D^2 \Phi) \Phi^+ \Phi^+ \\
& + x_{21,22} \epsilon^{\alpha\beta} \left( \eta \sigma \Lambda \Lambda^k \right)^l \theta^4 \partial_k (D_\beta \Phi) \partial_l (D_\alpha \Phi) \Phi^+ \Phi^+ \\
& + x_{21,23} \Lambda^{kl} \theta^4 \partial_k \partial_l \Phi (D^2 \Phi) \Phi^+ \Phi^+ \\
& + \frac{1}{2} (i x_{21,26}) \eta_{kl} \left( \bar{\sigma}^l \right)^{\dot{\alpha}\beta} \left( \eta \sigma \Lambda \Lambda^k \right)^n \theta^4 (D_\beta \Phi) \partial_n (D^2 \Phi) \Phi^+ (\bar{D}_{\dot{\alpha}} \Phi^+) \\
& + \frac{1}{2} (-i x_{21,26}) \eta_{kl} \left( \bar{\sigma}^l \right)^{\dot{\alpha}\beta} \left( \eta \sigma \Lambda \Lambda^k \right)^n \theta^4 \partial_n (D_\beta \Phi) (D^2 \Phi) \Phi^+ (\bar{D}_{\dot{\alpha}} \Phi^+) \\
& + x_{21,26} \epsilon^{\alpha\beta} \epsilon^{\zeta\iota} \Lambda^k_\beta \Lambda^l_\iota \theta^4 \partial_k (D_\alpha \Phi) \partial_l (D_\zeta \Phi) \Phi^+ \Phi^+ \\
& + x_{21,27} \epsilon^{\alpha\beta} \Lambda^{kl} \theta^4 \partial_k (D_\alpha \Phi) \partial_l (D_\beta \Phi) \Phi^+ \Phi^+ \\
& + (\Lambda^2 (-i y_{21,35} - i y_{21,36} - y_{21,37}) + \sigma \Lambda \Lambda (-i z_{21,35} - i z_{21,36} - z_{21,37})) \eta^{kl} \epsilon^{\alpha\beta} \theta^4 \partial_k (D_\alpha \Phi) \partial_l (D_\beta \Phi) \Phi^+ \Phi^+ \\
& + (\Lambda^2 (-i y_{21,35} + i y_{21,36} + y_{21,39}) + \sigma \Lambda \Lambda (-i z_{21,35} + i z_{21,36} + z_{21,39})) \theta^4 \square \Phi (D^2 \Phi) \Phi^+ \Phi^+ \\
& + \left( \Lambda^2 \left( -\frac{1}{16} i y_{21,35} - \frac{1}{16} i y_{21,36} \right) + \sigma \Lambda \Lambda \left( -\frac{1}{16} i z_{21,35} - \frac{1}{16} i z_{21,36} \right) \right) \epsilon^{\dot{\alpha}\dot{\beta}} \theta^4 (D^2 \Phi) (D^2 \Phi) (\bar{D}_{\dot{\alpha}} \Phi^+) (\bar{D}_{\dot{\beta}} \Phi^+) \\
& + \left( \Lambda^2 \left( \frac{y_{21,32}}{16} - \frac{1}{16} i y_{21,35} - \frac{1}{16} i y_{21,36} + \frac{y_{21,38}}{16} \right) + \sigma \Lambda \Lambda \left( \frac{z_{21,32}}{16} - \frac{1}{16} i z_{21,35} - \frac{1}{16} i z_{21,36} + \frac{z_{21,38}}{16} \right) \right) \theta^4 (D^2 \Phi) (D^2 \Phi) \Phi^+ (\bar{D}^2 \Phi^+) \\
& + (\Lambda^2 y_{21,32} + \sigma \Lambda \Lambda z_{21,32}) \epsilon^{\alpha\beta} \theta^4 (D_\alpha \Phi) (D_\beta \Phi) \Phi^+ \square \Phi^+ \\
& + \left( \Lambda^2 (2i y_{21,36} + 2y_{21,39}) + \sigma \Lambda \Lambda \left( -\frac{z_{21,34}}{2} + i z_{21,35} + i z_{21,36} + 2z_{21,39} \right) \right) \epsilon^{\alpha\beta} \theta^4 (D_\beta \Phi) \square (D_\alpha \Phi) \Phi^+ \Phi^+ \\
& + (\Lambda^2 (2i y_{21,35} - 2i y_{21,36}) + \sigma \Lambda \Lambda z_{21,34}) \left( \sigma^{kl} \right)^{\alpha\beta} \theta^4 \partial_k (D_\alpha \Phi) \partial_l (D_\beta \Phi) \Phi^+ \Phi^+ \\
& + (\Lambda^2 y_{21,35} + \sigma \Lambda \Lambda z_{21,35}) \left( \bar{\sigma}^k \right)^{\dot{\alpha}\beta} \theta^4 \partial_k (D_\beta \Phi) (D^2 \Phi) \Phi^+ (\bar{D}_{\dot{\alpha}} \Phi^+) \\
& + (\Lambda^2 y_{21,36} + \sigma \Lambda \Lambda z_{21,36}) \left( \bar{\sigma}^k \right)^{\dot{\alpha}\beta} \theta^4 (D_\beta \Phi) \partial_k (D^2 \Phi) \Phi^+ (\bar{D}_{\dot{\alpha}} \Phi^+) \\
& + (\Lambda^2 y_{21,37} + \sigma \Lambda \Lambda z_{21,37}) \eta^{kl} \theta^4 \partial_l \Phi \partial_k (D^2 \Phi) \Phi^+ \Phi^+ \\
& + (\Lambda^2 y_{21,38} + \sigma \Lambda \Lambda z_{21,38}) \theta^4 \Phi (D^2 \Phi) \Phi^+ \square \Phi^+ \\
& + (\Lambda^2 y_{21,39} + \sigma \Lambda \Lambda z_{21,39}) \theta^4 \Phi \square (D^2 \Phi) \Phi^+ \Phi^+, \\
B_{22} &= (\Lambda^2 y_{22,2} + \sigma \Lambda \Lambda z_{22,2}) \theta^4 \Phi \square (D^2 \Phi) \\
&+ (\Lambda^2 y_{22,2} + \sigma \Lambda \Lambda z_{22,2}) \epsilon^{\alpha\beta} \theta^4 (D_\beta \Phi) \square (D_\alpha \Phi), \\
B_{23} &= (-8i \Lambda^2 y_{23,3} - 8i \sigma \Lambda \Lambda z_{23,3}) \left( \bar{\sigma}^k \right)^{\dot{\alpha}\beta} \theta^4 (D_\beta \Phi) \partial_k (\bar{D}_{\dot{\alpha}} \Phi^+) \\
&+ (16 \Lambda^2 y_{23,3} + 16 \sigma \Lambda \Lambda z_{23,3}) \theta^4 \Phi \square \Phi^+ \\
&+ (\Lambda^2 y_{23,3} + \sigma \Lambda \Lambda z_{23,3}) \theta^4 (D^2 \Phi) (\bar{D}^2 \Phi^+), \\
B_{24} &= (-8i \Lambda^2 y_{24,3} - 8i \sigma \Lambda \Lambda z_{24,3}) \left( \bar{\sigma}^k \right)^{\dot{\alpha}\beta} \theta^4 (D_\beta \Phi) \Phi^+ \partial_k (\bar{D}_{\dot{\alpha}} \Phi^+) \\
&+ (16 \Lambda^2 y_{24,3} + 16 \sigma \Lambda \Lambda z_{24,3}) \theta^4 \Phi \Phi^+ \square \Phi^+ \\
&+ (\Lambda^2 y_{24,3} + \sigma \Lambda \Lambda z_{24,3}) \theta^4 (D^2 \Phi) \Phi^+ (\bar{D}^2 \Phi^+),
\end{aligned}$$

$$\begin{aligned}
B_{25} &= (-8i\Lambda^2 y_{25,3} - 8i\sigma\Lambda\Lambda z_{25,3}) (\bar{\sigma}^k)^{\dot{\alpha}\beta} \theta^4 (D_\beta \Phi) \Phi^+ \Phi^+ \partial_k (\bar{D}_{\dot{\alpha}} \Phi^+) \\
&\quad + (16\Lambda^2 y_{25,3} + 16\sigma\Lambda\Lambda z_{25,3}) \theta^4 \Phi \Phi^+ \Phi^+ \square \Phi^+ \\
&\quad + (\Lambda^2 y_{25,3} + \sigma\Lambda\Lambda z_{25,3}) \theta^4 (D^2 \Phi) \Phi^+ \Phi^+ (\bar{D}^2 \Phi^+) , \\
B_{26} &= \left( -\frac{1}{2} i \Lambda^2 y_{26,3} - \frac{1}{2} i \sigma \Lambda \Lambda z_{26,3} \right) (\bar{\sigma}^k)^{\dot{\alpha}\beta} \theta^4 (D_\beta \Phi) \Phi^+ \Phi^+ \Phi^+ \partial_k (\bar{D}_{\dot{\alpha}} \Phi^+) \\
&\quad + \left( \frac{1}{16} \Lambda^2 y_{26,3} + \frac{1}{16} \sigma \Lambda \Lambda z_{26,3} \right) \theta^4 (D^2 \Phi) \Phi^+ \Phi^+ \Phi^+ (\bar{D}^2 \Phi^+) \\
&\quad + (\Lambda^2 y_{26,3} + \sigma\Lambda\Lambda z_{26,3}) \theta^4 \Phi \Phi^+ \Phi^+ \Phi^+ \square \Phi^+ , \\
B_{27} &= \left( \Lambda^2 \left( \frac{3y_{27,3}}{2} + y_{27,4} \right) + \sigma\Lambda\Lambda \left( \frac{3z_{27,3}}{2} + z_{27,4} \right) \right) \theta^4 \Phi (D^2 \Phi) \square (D^2 \Phi) \\
&\quad + (\Lambda^2 (y_{27,3} + y_{27,4}) + \sigma\Lambda\Lambda (z_{27,3} + z_{27,4})) \eta^{kl} \theta^4 \Phi \partial_k (D^2 \Phi) \partial_l (D^2 \Phi) \\
&\quad + (\Lambda^2 y_{27,3} + \sigma\Lambda\Lambda z_{27,3}) \epsilon^{\alpha\beta} \theta^4 (D_\beta \Phi) \square (D_\alpha \Phi) (D^2 \Phi) \\
&\quad + (\Lambda^2 y_{27,4} + \sigma\Lambda\Lambda z_{27,4}) \eta^{kl} \epsilon^{\alpha\beta} \theta^4 \partial_k (D_\alpha \Phi) \partial_l (D_\beta \Phi) (D^2 \Phi) , \\
B_{28} &= \left( -\frac{x_{28,3}}{2} + x_{28,4} \right) (\eta\sigma\Lambda\Lambda^k)^l \theta^4 \Phi \partial_k (D^2 \Phi) \partial_l (D^2 \Phi) \\
&\quad + (3x_{28,3} - 2x_{28,4} - 2x_{28,5}) \epsilon^{\alpha\beta} \eta_{kl} (\sigma\Lambda^{ln})^{o\zeta} \Lambda^k{}_\beta \theta^4 \partial_n (D_\alpha \Phi) \partial_o (D_\zeta \Phi) (D^2 \Phi) \\
&\quad + x_{28,3} \epsilon^{\alpha\beta} (\eta\sigma\Lambda\Lambda^k)^l \theta^4 (D_\beta \Phi) \partial_k \partial_l (D_\alpha \Phi) (D^2 \Phi) \\
&\quad + x_{28,4} (\eta\sigma\Lambda\Lambda^k)^l \theta^4 \Phi (D^2 \Phi) \partial_k \partial_l (D^2 \Phi) \\
&\quad + x_{28,5} \epsilon^{\alpha\beta} (\eta\sigma\Lambda\Lambda^k)^l \theta^4 \partial_k (D_\beta \Phi) \partial_l (D_\alpha \Phi) (D^2 \Phi) , \\
B_{29} &= (-2x_{29,3} + 2x_{29,4}) \epsilon^{\alpha\beta} \Lambda^{kl} \theta^4 (D_\beta \Phi) \partial_l \partial_k (D_\alpha \Phi) (D^2 \Phi) \\
&\quad + (-6x_{29,3} + 4x_{29,4} + 2x_{29,5}) \epsilon^{\alpha\beta} \epsilon^{\zeta\iota} \Lambda^k{}_\beta \Lambda^l{}_\iota \theta^4 \partial_k (D_\alpha \Phi) \partial_l (D_\zeta \Phi) (D^2 \Phi) \\
&\quad + x_{29,3} \Lambda^{kl} \theta^4 \Phi \partial_k (D^2 \Phi) \partial_l (D^2 \Phi) \\
&\quad + x_{29,4} \Lambda^{kl} \theta^4 \Phi (D^2 \Phi) \partial_l \partial_k (D^2 \Phi) \\
&\quad + x_{29,5} \epsilon^{\alpha\beta} \Lambda^{kl} \theta^4 \partial_k (D_\alpha \Phi) \partial_l (D_\beta \Phi) (D^2 \Phi) , \\
B_{30} &= (\Lambda^2 y_{30,1} + \sigma\Lambda\Lambda z_{30,1}) \theta^4 (D^2 \Phi) , \\
B_{31} &= x_{31,1} (\sigma\Lambda\Lambda^{kl})^{no} \theta^4 \Phi^+ \partial_k \partial_n \Phi^+ \partial_l \partial_o \Phi^+ , \\
B_{32} &= (\Lambda^2 y_{32,1} + \sigma\Lambda\Lambda z_{32,1}) (\sigma^{kl})^{\alpha\beta} \theta^4 \partial_k (D_\alpha \Phi) \partial_l (D_\beta \Phi) (D^2 \Phi) , \\
B_{33} &= x_{33,1} \eta^{kl} (\eta\sigma\Lambda\Lambda^n)^o \theta^4 \Phi^+ \partial_l \partial_n \Phi^+ \partial_k \partial_o \Phi^+ , \\
B_{34} &= x_{34,1} (\eta\sigma\Lambda\Lambda^k)^l \theta^4 \Phi^+ \Phi^+ \partial_k \Phi^+ \partial_l \Phi^+ , \\
B_{35} &= x_{35,1} (\eta\sigma\Lambda\Lambda^k)^l \theta^4 \Phi^+ \Phi^+ \Phi^+ \partial_k \partial_l \Phi^+ , \\
B_{36} &= x_{36,1} \Lambda^{kl} \eta_{ln} (\sigma^{no})^{\alpha\beta} \theta^4 \partial_k (D_\alpha \Phi) \partial_o (D_\beta \Phi) (D^2 \Phi) , \\
B_{37} &= x_{37,1} \epsilon^{\alpha\beta} \epsilon^{\zeta\iota} \epsilon^{klno} \eta_{mp} \eta_{oq} \Lambda^p{}_\beta \Lambda^q{}_\iota \theta^4 \partial_k (D_\alpha \Phi) \partial_l (D_\zeta \Phi) (D^2 \Phi) , \\
B_{38} &= (\Lambda^2 y_{38,1} + \sigma\Lambda\Lambda z_{38,1}) \theta^4 (D^2 \Phi) (D^2 \Phi) , \\
B_{39} &= (\Lambda^2 y_{39,1} + \sigma\Lambda\Lambda z_{39,1}) \theta^4 \Phi^+ \square \Phi^+ , \\
B_{40} &= (\Lambda^2 y_{40,1} + \sigma\Lambda\Lambda z_{40,1}) \theta^4 (D^2 \Phi) (D^2 \Phi) \Phi^+ , \\
B_{41} &= (\Lambda^2 y_{41,1} + \sigma\Lambda\Lambda z_{41,1}) \theta^4 \Phi^+ \Phi^+ \square \Phi^+ , \\
B_{42} &= x_{42,1} (\eta\sigma\Lambda\Lambda^k)^l \theta^4 (D^2 \Phi) \partial_k \partial_l (D^2 \Phi) , \\
B_{43} &= x_{43,1} (\eta\sigma\Lambda\Lambda^k)^l \theta^4 \Phi^+ \square \partial_k \partial_l \Phi^+ , \\
B_{44} &= x_{44,1} \Lambda^{kl} \theta^4 (D^2 \Phi) \partial_l \partial_k (D^2 \Phi) , \\
B_{45} &= x_{45,1} \Lambda^{kl} \theta^4 \Phi^+ \square \partial_k \partial_l \Phi^+ , \\
B_{46} &= (\Lambda^2 y_{46,1} + \sigma\Lambda\Lambda z_{46,1}) \eta^{kl} \eta^{no} \theta^4 \Phi^+ \partial_k \partial_n \Phi^+ \partial_l \partial_o \Phi^+ , \\
B_{47} &= x_{47,1} \Lambda^{kl} \theta^4 \Phi^+ \Phi^+ \partial_k \Phi^+ \partial_l \Phi^+ , \\
B_{48} &= x_{48,1} \Lambda^{kl} \theta^4 \Phi^+ \Phi^+ \Phi^+ \partial_k \partial_l \Phi^+ ,
\end{aligned}$$

$$\begin{aligned}
B_{49} &= (\Lambda^2 y_{49,1} + \sigma \Lambda \Lambda z_{49,1}) \theta^4 (D^2 \Phi) \Phi^+, \\
B_{50} &= (\Lambda^2 y_{50,1} + \sigma \Lambda \Lambda z_{50,1}) \theta^4 (D^2 \Phi) \Phi^+ \Phi^+, \\
B_{51} &= (\Lambda^2 y_{51,1} + \sigma \Lambda \Lambda z_{51,1}) \theta^4 (D^2 \Phi) \Phi^+ \Phi^+ \Phi^+, \\
B_{52} &= x_{52,1} \left( \eta \sigma \Lambda \Lambda^k \right)^l \theta^4 \Phi^+ \square \Phi^+ \partial_k \partial_l \Phi^+, \\
B_{53} &= x_{53,1} \left( \eta \sigma \Lambda \Lambda^k \right)^l \theta^4 (D^2 \Phi) \partial_k \Phi^+ \partial_l \Phi^+, \\
B_{54} &= x_{54,1} \Lambda^{kl} \theta^4 (D^2 \Phi) \partial_k \Phi^+ \partial_l \Phi^+, \\
B_{55} &= x_{55,1} \left( \eta \sigma \Lambda \Lambda^k \right)^l \theta^4 (D^2 \Phi) \partial_k \partial_l \Phi^+, \\
B_{56} &= x_{56,1} \Lambda^{kl} \theta^4 (D^2 \Phi) \partial_k \partial_l \Phi^+, \\
B_{57} &= x_{57,1} \left( \eta \sigma \Lambda \Lambda^k \right)^l \theta^4 (D^2 \Phi) \Phi^+ \partial_k \partial_l \Phi^+, \\
B_{58} &= x_{58,1} \Lambda^{kl} \theta^4 (D^2 \Phi) \Phi^+ \partial_k \partial_l \Phi^+, \\
B_{59} &= x_{59,1} \left( \eta \sigma \Lambda \Lambda^k \right)^l \theta^4 (D^2 \Phi) \square \partial_k \partial_l \Phi^+, \\
B_{60} &= x_{60,1} \Lambda^{kl} \theta^4 (D^2 \Phi) \square \partial_k \partial_l \Phi^+, \\
B_{61} &= x_{61,1} \left( \eta \sigma \Lambda \Lambda^k \right)^l \theta^4 \Phi^+ \Phi^+ \square \partial_k \partial_l \Phi^+, \\
B_{62} &= x_{62,1} \eta^{kl} \Lambda^{no} \theta^4 \Phi^+ \partial_k \partial_n \Phi^+ \partial_l \partial_o \Phi^+, \\
B_{63} &= x_{63,1} \Lambda^{kl} \theta^4 \Phi^+ \Phi^+ \square \partial_k \partial_l \Phi^+, \\
B_{64} &= x_{64,1} \Lambda^{kl} \theta^4 \Phi^+ \square \Phi^+ \partial_k \partial_l \Phi^+, \\
B_{65} &= (\Lambda^2 y_{65,1} + \sigma \Lambda \Lambda z_{65,1}) \theta^4 \Phi^+ \square \Phi^+ \square \Phi^+, \\
B_{66} &= (\Lambda^2 y_{66,1} + \sigma \Lambda \Lambda z_{66,1}) \eta^{kl} \theta^4 \Phi^+ \Phi^+ \partial_k \Phi^+, \partial_l \Phi^+ \\
B_{67} &= (\Lambda^2 y_{67,1} + \sigma \Lambda \Lambda z_{67,1}) \theta^4 (D^2 \Phi) \square \square \Phi^+, \\
B_{68} &= (\Lambda^2 y_{68,1} + \sigma \Lambda \Lambda z_{68,1}) \theta^4 \Phi^+ \Phi^+ \square \square \Phi^+, \\
B_{69} &= (\Lambda^2 y_{69,1} + \sigma \Lambda \Lambda z_{69,1}) \theta^4 \Phi^+ \Phi^+ \Phi^+ \square \Phi^+, \\
B_{70} &= (\Lambda^2 y_{70,1} + \sigma \Lambda \Lambda z_{70,1}) \eta^{kl} \theta^4 (D^2 \Phi) \partial_k \Phi^+, \partial_l \Phi^+ \\
B_{71} &= (\Lambda^2 y_{71,1} + \sigma \Lambda \Lambda z_{71,1}) \theta^4 (D^2 \Phi) \square (D^2 \Phi), \\
B_{72} &= (\Lambda^2 y_{72,1} + \sigma \Lambda \Lambda z_{72,1}) \theta^4 \Phi^+ \square \square \Phi^+, \\
B_{73} &= (\Lambda^2 y_{73,1} + \sigma \Lambda \Lambda z_{73,1}) \theta^4 (D^2 \Phi) \square \Phi^+, \\
B_{74} &= (\Lambda^2 y_{74,1} + \sigma \Lambda \Lambda z_{74,1}) \theta^4 (D^2 \Phi) \Phi^+ \square \Phi^+.
\end{aligned} \tag{B2}$$

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